Impact of Finite Life Cycle to a Consignment Stocking Supply Chain with Uncertain Demand

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ABSTRACT

There are usually two reasons that force a supply chain to adopt the consignment stocking policy: the high demand variance and the high risk of obsolescence. This research aims to study the influences of these two issues to a consignment stocking integrated supply chain. To better reflect the real situation in the engineering practice, the buyer's warehouse limitation and a reducible lead time is also considered. A five-variable mixed-integer optimization model that aimed to minimize the annual joint total expected cost (JTEC) is developed under the general assumption that the production rate, the number of shipments and the shipping quantities are all fixed. To distinguish the Consignment Stocking policy from traditional ones, the holding cost is considered to consist of a storage component and a financial component. Due to the complexity of the objective function, traditional mathematical methods are cumbersome. As a result, closed-form solutions for the model are not provided. Instead, a novel doubly-hybrid meta-heuristic algorithm is employed to solve the problem. A numerical example is used to illustrate the solution procedure and the efficiency of the doubly-hybrid meta-heuristic algorithm.

1. INTRODUCTION

Consignment Stocking (CS) is a novel approach of cooperation between suppliers and customers and has become popular in variety of industries (Chen and Liu, 2008). The risk of obsolescence is important to the success of a buyer-vendor integrated system taking the CS policy. Failure to consider the impact of obsolescence, resulting in unsold products, will decrease the profit of the supply chain. However, it is rare in the literature that that factor is considered, especially in the context of an integrated CS system. Traditionally, the risk of obsolescence is borne solely by the buyer of a two-echelon supply chain. On the contrary, under a CS scenario, this risk is shared by both the vendor and the buyer, since the unsold products remain owned by the vendor. As a result, the replenishment policies of both the vendor and the buyer have to be modified to accommodate this risk. In order to provide the decision makers with such a system the optimal decisions against the obsolescence risk, a four-factor CS model that considers obsolescence, variability in demand rate, buyer's space limitation, and controllable lead time, is developed in this paper. The objective is to jointly decide the optimal ordering size, number of shipments within each production cycle, the number of delay shipments within each cycle, the lead time, and the safety stock, that minimize the annual joint total expected cost (JTEC) of the system.

Under a traditional CS policy, either the vendor keeps most of the inventory (when the buyer's unit inventory cost is much higher than that of the vendors) while maintaining a minimum amount of inventory in the buyer's warehouse, or the buyer stores the majority of the products (when the buyer's unit inventory cost is lower to the vendor), keeping the lower amount at the vendor's site. This is defined by Yi and Sarker (2013a, 2013b) as the CS ($k = 0$) and CS ($k = n - 1$) policy. The four-factor CS model to be developed here need to consider several system constraints.

Firstly, the buyer may want to place a space limitation to each of his/her supplier for each of the product. When there is an upper limit capacity in the space, the vendor cannot put as much inventory as s/he wants to the buyer. In the beginning, when the buyer's space limitation is not reached, the vendor is obliged, based on a CS agreement, to keep the buyer's inventory above a certain safety level. Toward this end, the products are shipped to the buyer in small quantities without having to wait until the up-time of each production cycle is ended. As a result, the buyer's inventory level

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gradually increases in the beginning of each cycle. In the meanwhile, the vendor maintains a minimum level of average inventory (equal to half the shipping size) during this period. However, when the inventory level in buyer's warehouse reaches its maximum level, I_{max}, which is close or equal to the buyer's space limitation, all later shipments from the vendor are delayed for a certain period so that the late arrival of a new shipment brings the buyer's inventory level back to I_{max}. As a result, the vendor's inventory level is forced to be increased to a certain level.

Secondly, the shipments are taking a certain period to reach the buyer, which incurs a shipping cost and a holding cost of the products in transit. However, the lead time can be reduced, often required by the buyer, with an extra charge, which is also shared by both the vendor and buyer under a long-term agreement. The lead time crushing cost is a function of both the time to be reduced and the quantity of the products to be shipped. The trade-off is that, while the shipping cost might be increased, the holding cost in transit is reduced, and the safety stock level is also reduced accordingly because of a shorter lead time.

The third constraint is that the demand rate may be uncertain. Based upon historical data; however, it may follow a stochastic distribution with known mean and variance. Because of the variation in demand, there might be back orders and/or some extra space may be needed in the buyer's warehouse. Both of the two cases cost the system an extra.

Finally, the product may be obsolescent sometime, which may occur at any time within the last production cycle. Thus, the last cycle may be incomplete. After that point, the unsold products and the material are considered to be lost. Hence, it incurs an obsolescence cost to the system.

The remainder of this paper is organized as follows. Section 2 defines parameters and assumptions used throughout the paper. The mathematical model is formulated in Section 3. Computational results are illustrated in Section 4. Conclusions are summarised in Section 5.

2. Notation and Assumptions

In order to develop the integrated models, the notations and assumptions used throughout this paper are given below:

2.1. Notations

A_v: vendor's batch setup cost (\$/setup),
A_b: Buyer's ordering cost (\$/order),
a_i: Maximum duration of the ith segment of lead time S_i (year),
b_i: Minimum duration of the ith segment of lead time S_i (year),
c_b: Unit backorder cost (\$/unit),
C_b: Expected annual backorder cost (\$/year),
c_{il}: Unit crashing cost for reducing one time unit of the ith segment of lead time S_i when the ordering quantity q is between q_{l-1} and q_l (\$/year),
c_o: Unit outsourcing cost (\$/unit),
C_o: Expected annual extra space cost (\$/year),
D: Yearly demand rate at the buyers' level (units/year), $D \sim N(\mu,\sigma)$.
$E(\bullet)$: Mathematical expectation of $\bullet$,
h: Annual unit holding cost (\$/unit/year),
H_v: Vendor's average annual inventory holding cost (\$/year),
h_v: Unit annual holding cost for inventory at the vendor (\$/unit/year), $h_v = h_v^s + h_v^f$,
h_v^f : Financial components of h_v (\$/unit/year), $h_v^f = rp_v$,
h_v^s : Storage component of h_v (\$/unit/year),
H_b: Buyer's average annual inventory holding cost (\$/year),
h_b: Unit annual holding cost for inventory at the buyer (\$/unit/year), $h_b = h_b^s + h_v^f$,
h_b^s : Storage component of h_b (\$/unit/year),
H_d: Average annual holding cost for inventory in transit (\$/year),
h_d: Unit annual holding cost in transit (\$/unit/year), $h_d = h_d^s + h_v^f$
h_d^s : Storage component of the holding cost of h_d (\$/unit/year),
r: Opportunity cost of capital (%/year),
I: Average inventory (units),
I_b: Buyer's average inventory (units),
I_{max}: Buyer's maximum inventory level (units),

I_s: Average system inventory (units),
I_v: Vendor's average inventory (units),
k: Number of delayed deliveries due to buyer's stock capacity,
L: Length of lead time (year), $L = \sum S_i$,
L_i: Lead time (year) where the ith component S_i was crashed to its minimum duration a_i,
n: Number of delivery operations per full production batch,
n^*: Number of full production cycles during the entire planning horizon, $n^* = \lfloor \mu T / nq \rfloor$,
P: Vendor's production rate (units/year),
p_v: Vendor's unit production cost ($/unit),
p_b: Buyer's unit purchasing cost ($/unit),
q: Size of each delivery or shipped lot (batch size $Q = nq$),
$R(q, L)$: Lead time crashing cost per replenishment cycle ($/shipment),
s: Safety factor,
S_i: ith segment of lead time (year), $a_i \le S_i \le b_i$,
s_s: Safety stock level,
t: Length of the last incomplete production cycle (year), $t = T - n^* \times nq / \mu$,
t_{li}^{I}: Lower bound of the *i*th scenario associated with Model I (year), where i is a positive integer less or equal to four.
t_{ui}^{I}: Upper bound of the *i*th scenario associated with Model I (year), where i is a positive integer less or equal to four.
t_{li}^{II}: Lower bound of the *i*th scenario associated with Model II (year), where i is a positive integer less or equal to five.
t_{ui}^{II}: Upper bound of the *i*th scenario associated with Model II (year), where i is a positive integer less or equal to five.
T: Item life period (year),
U: Space limitation placed by the buyer to the vendor (units),
μ: Expectation value of annual demand rate D (units/year), $\mu = \int_{-\infty}^{+\infty} Df(D)dD$,
σ: Standard deviation of annual demand rate D (units),
v: Number of price segments associated with ordering quantity,
x^+: Maximum value of x and 0, i.e. $x^+ = \max\{x, 0\}$,
X_1: The random demand during the period (q/P + L), having a mean $\mu(q/P+L)$ and standard deviation $\sigma\sqrt{q/P+L}$,
X_2: The random demand during the period q/P, having a mean $q\mu/P$ and standard deviation $\sigma\sqrt{q/P}$,
X_3: The random demand during the period q/μ , having a mean q and standard deviation $\sigma\sqrt{q/\mu}$,
X_4: A random variable associated with the four-factor CS model, i.e., the during the period $(n-k)q/\mu - (n-k)q/P - L$, having a mean $(n-k)q \;\; -(n-k)q\mu/P - \mu L$ and standard deviation $\sigma\sqrt{(n-k)q/\mu - (n-k)q/P - L}$,
X_5: The random demand during the last incomplete production cycle t, having a mean μt and standard deviation $\sigma\sqrt{t}$,
X_6: The random demand during the period $t - L - q/P - \lfloor (t-L-q/P)P/q \rfloor q/P$, having a standard deviation $\sigma\sqrt{t - L - q/P - \lfloor (t-L-q/P)P/q \rfloor q/P}$ and a mean $\mu(t-L-q/P) \;\; -\mu\lfloor (t-L-q/P)P/q \rfloor \, q/P$,
X_7: A random variable associated with scenarios 3 and 4 of the four-factor CS model I, i.e., the demand during the period $t - L - (n-k)q/P - \lfloor [t - L - (n-k)q/P]\mu/q \rfloor q/\mu$, having a mean $\mu[t - L - (n-k)q/P] - \lfloor [t - L - (n-k)q/P]\mu/q \rfloor q$ and a standard deviation $\sigma\sqrt{t - L - (n-k)q/P - \lfloor [t - L - (n-k)q/P]\mu/q \rfloor q/\mu}$,
X_8: The random demand during the period $(n-k+1)q/\mu - (n-k-1)q/P - U/\mu \;\; - s\sigma\sqrt{L+q/P}\big/\mu$, having a mean $(n-k+1)q - \mu(n-k-1)q/P - U - s\sigma\sqrt{L+q/P}$ and a standard deviation $\sigma\sqrt{(n-k+1)q/\mu - (n-k-1)q/P - U/\mu - s\sigma\sqrt{L+q/P}\big/\mu}$,
X_9: The random demand during the period $U/\mu - q/P - L$, which is the duration of the first delayed shipment in the modified four-factor CS model. X_9 has a mean $U - \mu q/P - \mu L$ and a standard deviation $\sigma\sqrt{U/\mu - q/P - L}$,
X_{10}: A random variable associated the modified four-factor CS model, i.e., the demand during the period $t - (n-k)q/P - L$, having a mean $\mu[t - (n-k)q/P - L]$ and a standard deviation $\sigma\sqrt{t - (n-k)q/P - L}$,
X_{11}: A random variable associated the modified four-factor CS model, i.e., the demand during the period $t - \lfloor (t - t_{l4}^{II})\mu/q \rfloor q/\mu$, having a mean $\mu t - \lfloor (t - t_{l4}^{II})\mu/q \rfloor q$ and a standard deviation $\sigma\sqrt{t - \lfloor (t - t_{l4}^{II})\mu/q \rfloor q/\mu}$.

2.2. ASSUMPTIONS

The following assumptions are necessary to the model developed in this paper:

(1) The yearly production rate P is finite.
(2) The demand of the buyer D follows a normal distribution with a mean μ and a standard deviation σ, i.e., $D \sim N(\mu, \sigma)$.
(3) The demand during lead-time L also follows normal distribution with a mean μL and a standard deviation $\sigma\sqrt{L}$.
(4) The inventory is continuously reviewed.
(5) The products are shipped to the buyer in n batches.
(6) Due to the demand uncertainty, shortage is allowed in the system and is backordered with a shortage cost.
(7) Also due to the demand uncertainty, extra inventories beyond the buyer's capacity are allowed. The extra products may be still stored in the buyer's warehouse but in a space reserved for other products/suppliers of the buyer or they may be stored by a third party. In both cases the vendor will be charged an extra penalty cost.
(8) Unlike most other researchers who consider the shipping cost to be a function of the guaranteed lead time, this study assume that it is an incremental function of both the guaranteed lead time and the shipment quantity.
(9) The predicted time between the beginning of the first production cycle and the date of obsolescence is deterministic (Persona et al, 2005, Battini et al, 2010a, Battini et al, 2010b).

3. MODEL FORMULATION

In this Section, the CS-*k* model considering four practical factors: (a) obsolescence (Obs), (b) controllable lead time (CLT), (c) buyer's capacity limitation (BCL), and (d) stochastic demand (SD) is developed according to the pattern of the system inventory, the vendor inventory, the inventory in transit, and the buyer inventory in a full production cycle of this model is shown in Figure 1. Specifically, the inventories in a full production cycle is shown in with dashed lines, whereas the inventory patterns during the last incomplete production cycle is shown with solid lines.

Unlike normal full production cycles, the last cycle may be incomplete due to the product obsolescence. To avoid further lost caused by the obsolescence, the vendor is assumed to cease its production at the moment when the obsolescence occurs. The remaining stocks are considered to be lost and will be cleared from the warehouses immediately. The in-transit inventory, if any, will be cleared when it reaches the buyer. Because the obsolescence can occur anytime within the last cycle, the inventory patterns can be categorized into four scenarios according to the time that the obsolescence may occur (Cases 1, 2, 3, and 4 in Figure 1). Specifically, Case 1 is the situation when the obsolescence occurs before the arrival of the first shipment. Case 2 reflects when the obsolescence occurs after the arrival of the first shipment, but before the maximum inventory level I_{max} is reached. Case 3 describes when obsolescence occurs after the beginning of the first delayed shipment, but before the end of the up-time $T_{up} = nq / P$. Case 4 illustrates when obsolescence occurs during the down-time. To save space, only case 3 are used to develop the four-factor CS model.

According to Valentini and Zavanella (2003), the buyer's ordering cost is zero under a CS policy. As the result, the joint total expected cost of the system, *JTEC*, can be written as the sum of the vendor's expected setup cost, the buyer's expected backorder cost, the buyer's expected extra space cost, the extra expected lead time crushing cost, the expected system holding cost, and the obsolescence cost. The calculations of all these costs are discussed below.

The vendor's expected annual setup cost C_s^v can be written as

$$C_s^v = \frac{A_v}{T}\left\lceil \frac{\mu T}{nq} \right\rceil. \tag{1}$$

The expected annual lead time crushing cost can be written as

$$C_L = \frac{R(q,L)}{T}\left\lceil \frac{\mu T}{q} \right\rceil = \frac{qc_{il}(L_{i-1} - L) + q\sum_{j=1}^{i-1} c_{jl}(b_j - a_j)}{T}\left\lceil \frac{\mu T}{q} \right\rceil, \tag{2}$$

where, $i = 1, 2, ..., m$, $l = 1, 2, ..., r$ for $q \in (q_{l-1}, q_l)$ and $L \in (L_i, L_{i-1})$.

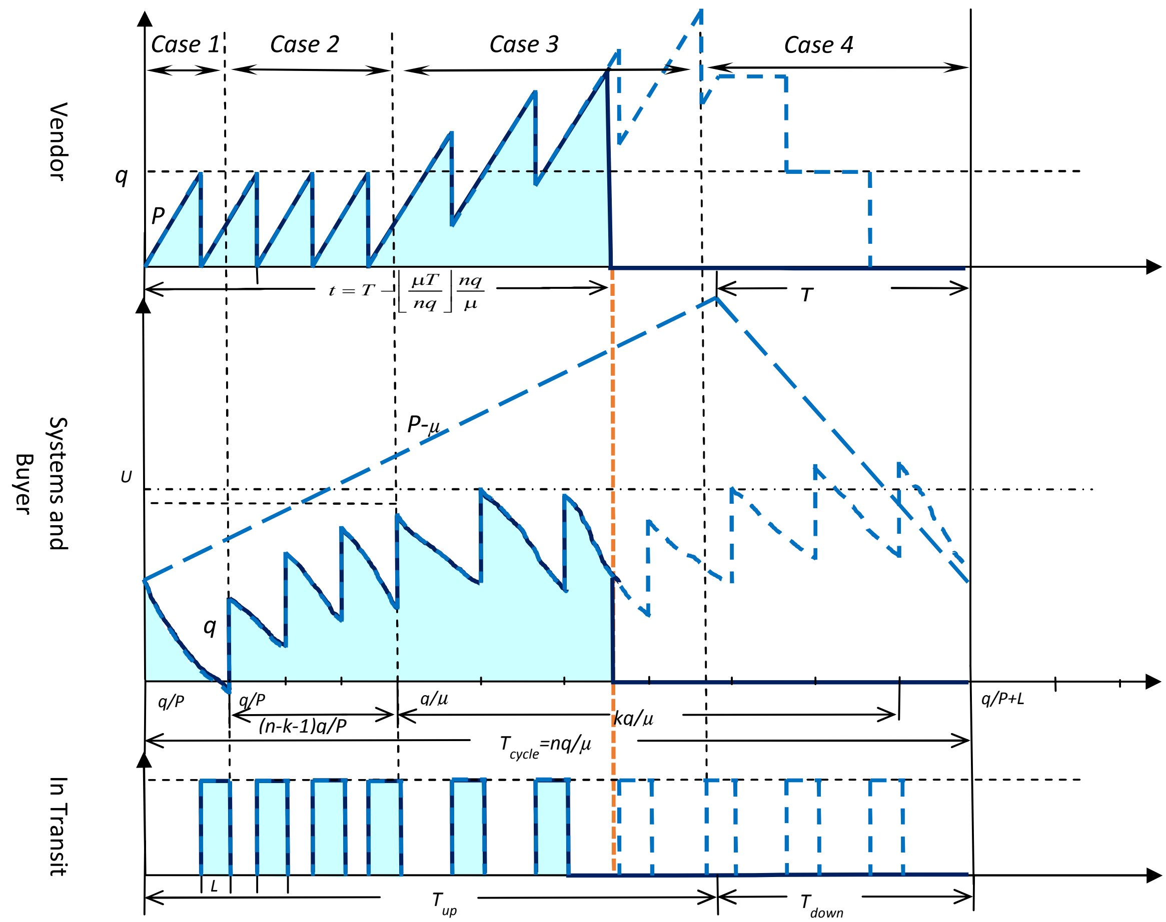

Figure 1. Composition of the inventory in the four-factor CS Model.

The expected annual holding cost incurred in the buyer's inventory H_b of case 3 can be written as:

$$H_b=(h_b^s+h_v^f)\ \Bigg\{\left[\frac{q\mu}{2P}+nq\frac{P-\mu}{2P}-q\frac{P-\mu}{nP}\frac{(k+1)k}{2}+s\sigma\sqrt{L+q/P}\right]n^*\frac{nq}{\mu}$$
$$+\frac{1}{2}\left(\frac{q\mu}{P}+\mu L+s\sigma\sqrt{L+\frac{q}{P}}\right)\left(L+\frac{q}{P}\right)+\frac{(n-k)(2P-\mu)q^2+(n-k)(n-k-1)(P-\mu)q^2}{2P^2}$$
$$+\frac{i'q}{\mu}\left[\left(n-k-\frac{1}{2}\right)q-\frac{(n-k-1)q\mu}{P}+s\sigma\sqrt{L+\frac{q}{P}}\right]+\left[(n-k)q-\frac{(n-k-1)q\mu}{P}+s\sigma\sqrt{L+\frac{q}{P}}\ -\frac{\mu}{2}\left(\tau'-\frac{i'q}{\mu}\right)\right]\left(\tau'-\frac{i'q}{\mu}\right)\Bigg\}\Big/T, \quad (3)$$

where, $n^*=\lfloor \mu T/nq\rfloor$, $\tau'=t-L-(n-k)q/P$, and $i'=\lfloor \tau'\mu/q\rfloor$.

The expected holding cost incurred in the vendor's warehouse H_v of case 3 is written as

$$H_v=(h_v^s+h_v^f)\ \Bigg\{\left[\frac{q\mu}{2P}+q\frac{P-\mu}{nP}\frac{(k+1)k}{2}\right]n^*\frac{nq}{\mu}+\frac{(n-k)q^2}{2P}+\frac{n'Pq^2}{2\mu^2}+\frac{n'(n'+1)(P-\mu)q^2}{2\mu^2}$$
$$+\frac{n'q(P-\mu)}{2\mu}\left[t-\frac{(n-k)q}{P}-\frac{n'q}{\mu}\right]+\frac{P}{2}\left[t-\frac{(n-k)q}{P}-\frac{n'q}{\mu}\right]^2\Bigg\}\Big/T, \quad (4)$$

where, $n^*=\lfloor \mu T/nq\rfloor$ and $n'=\left\lfloor \frac{t\mu}{q}-\frac{(n-k)\mu}{P}\right\rfloor-1$.

The expected annual holding cost in transit H_d for case 3 is given by:

$$H_d=(h_d^s+h_v^f)\left[nqLn^*+\min\left(n,n-k+\left\lfloor\frac{t\mu}{q}-\frac{n-k}{P}\mu\right\rfloor\right)qL\right]\Big/T, \quad (5)$$

The expected annual backorder cost C_b of case 3 can be written as:

$$C_b = c_b\left\{(n^*+1)E\left(X_1-\frac{q\mu}{P}-\mu L-s\sigma\sqrt{L+\frac{q}{P}}\right)^+ +(n^*+1)\sum_{i=0}^{n-k-2}E\left(X_2-(i+1)q+\frac{iq\mu}{P}-s\sigma\sqrt{L+\frac{q}{P}}\right)^+\right.$$

$$+\left[n^*(k-1)+\left\lfloor\frac{\tau'\mu}{q}\right\rfloor\right]E\left(X_3-(n-k)q+(n-k-1)\frac{q\mu}{P}-s\sigma\sqrt{L+\frac{q}{P}}\right)^+ +E\left(X_4-(n-k)q+(n-k-1)\frac{q\mu}{P}-s\sigma\sqrt{L+\frac{q}{P}}\right)^+$$

$$\left.+E\left(X_7-(n-k)q+(n-k-1)\frac{q\mu}{P}-s\sigma\sqrt{L+\frac{q}{P}}\right)^+\right\}\Bigg/T\,, \tag{6}$$

where, $n^*=\lfloor \mu T/nq\rfloor$ and $\tau' = t-L-(n-k)q/P$.

The expected annual extra space cost C_o of case 3 can be given as:

$$C_o=\frac{c_o}{T}\left(kn^*+\frac{\tau'\mu}{q}\right)E\left[(n-k+1)q-U-(n-k-1)q\mu/P+s\sigma\sqrt{L+\frac{q}{P}}-X_3\right]^+, \tag{7}$$

The expected annual obsolescence cost C_{ob} of case 3 can be calculated as:

$$C_{ob}=p_v\left[\frac{(P-\mu)q}{\mu}\left\lfloor\frac{\mu t}{q}-\frac{(n-k)\mu}{P}\right\rfloor+P\left(t-\frac{n-k}{P}q-\left\lfloor\frac{\mu t}{q}-\frac{(n-k)\mu}{P}\right\rfloor\frac{q}{\mu}\right)\right]$$

$$+p_b\left[(n-k)q-(n-k-1)\frac{q\mu}{P}+s\sigma\sqrt{L+\frac{q}{P}}-\mu\left(\tau'-\left\lfloor\frac{\tau'\mu}{q}\right\rfloor\frac{q}{\mu}\right)\right]+p_b q\left\lceil\frac{Max\left[0,t-t_{l3}^I-\left\lfloor\left(t-t_{l3}^I\right)\mu/q\right\rfloor q/\mu-q/\mu+L\right]}{L}\right\rceil, \tag{8}$$

Finally, the annual joint total expected cost $JTEC(q,n,k,s,L)$ for given $L\in(L_i,L_{i-1})$ and $q\in(q_{l-1},q_l)$ can be written as

$$JTEC(q,n,k,s,L)= C_s^v + C_L+H_v+H_d+H_b+C_b+C_o+C_{ob}\,. \tag{9}$$

The Four-factor CS model can, therefore, be written as:

Min $JTEC(q,n,k,s,L)$ (10)

Subject to: $(n-k)q-(n-k-1)q\mu/P+s\sigma\sqrt{L+q/P}\le U$, (11)

$k\le n$, (12)

$(n-k)q/P+L+kq/\mu\le nq/\mu$, (13)

q, n, k, L are positive and integers. (14)

The problem is to jointly decide the optimal ordering quantity *q*, safety factor *s*, number of shipments within a production cycle *n*, number of delayed shipments *k*, and lead time *L* that minimize the *JTEC* as expressed by Equation (10), under the constraints in (11) – (14). In which, the constraint in Equation (11) ensures that the buyer's space limitation is greater than the buyer's potential maximum inventory level, and the constraint in formula (13) ensures the arrival of all shipments to the buyer within each full production cycle.

The model developed here is a constrained nonlinear mixed-integer optimization problem. Due to its complexity, the traditional optimization methods are cumbersome [Persona, et al. (2005), Battini, et al. (2010a, 2010b)]. A newly developed doubly-hybrid meta-heuristic algorithm (DHMHA) (Yi, et al., 2013), which has been shown by Yi and Sarker (2013b, and 2013c) to be a satisfactory tool to solve complicated inventory models with multiple variables, is adopted in the next Section to locate the solutions. Also, to perform a numerical study, the normal distribution is adopted for the demand rate so that the expected backorder cost and the expected extra space cost can be evaluated.

4. Computational Results

In this Section, a numerical example is used to demonstrate the solution process. The outcomes of the numerical example, found by the two doubly-hybrid algorithms, are compared to that of an exhaustive search algorithm (ESA) to verify that the solutions found by the doubly-hybrid are global optimum. All the algorithms were coded in Matlab and were executed on a HP Pavilion Dv8 notebook PC with an Intel® QuadCore i7 CPU and Q 720@ 1.6 GHz processor. For each setting of the test parameters, 10 runs were made by both doubly-hybrid algorithms (PSO+IHS+HJ and MDE'+IHS+HJ). A run is declared as 'successful' when the global optimum (or the best-known) was found within 10^{-6} error.

A numerical example is framed here to illustrate the optimal solutions (ESA) along with PSO+IHS+HJ and MDE'+IHS+HJ. Most of the values of the parameters used in this example are adopted from Braglia and Zavanella

(2003), and Huang and Chen (2009). The lead time consists of three segment the composition of which is shown on Table 1 [the original time data are in days].

Table 1. The composition of the lead time components.

Lead time component i	Normal duration b_i (years)	Minimum duration a_i (years)
1	20/365=0.05479	6/365=0.01644
2	20/365=0.05479	6/365=0.01644
3	16/365=0.04384	9/365=0.02466

Similar as in Yi and Sarker (2013c), the unit lead time crushing cost in this example is a function of both the reduced period and the shipping size. A unified quantity discount is used to represent the relationship between the lead time crushing cost and the quantity. Table 2 illustrates the detailed composition of this cost.

Table 2. The composition of the unit lead time crushing cost $c_{ij.}$

	$q \geq 100$	$100 > q \geq 20$	$q < 20$
Lead time component i	c_{i1}**(\$/unit/year)**	c_{i2}**(\$/unit/year)**	c_{i3}**(\$/unit/year)**
1	(0.8)(0.1)(365)=29.2	(0.9)(0.1)(365)=32.85	(1)(0.1)(365)=36.5
2	(0.8)(1.2)(365)=350.4	(0.9)(1.2)(365)=394.2	(1)(1.2)(365)=438
3	(0.8)(5.0)(365)=1,460	(0.9)(5.0)(365)=1,642.5	(1)(5.0)(365)=1,825

The values of other parameters are: $A_v = \$400$ /setup, $A_b = \$25$ /order, $\mu = 1000$ units/year, $\sigma = 100$ units, $r = 10\%$, $P = 3200$ units/year, p_v = \$20/unit, $h_v^s = \$3$ /unit/year, $h_b^s = \$1.50$ /unit/year, $h_d^s = \$4$ /unit/year, c_o = \$10/unit, c_b = \$50/unit, and U = 150 units.

In the model developed in this study, all decision variables are integers except for the safety factor s, which is continuous. Two decimal values are allowed for s so that the ESA can be used to determine the optimal solutions and verify whether the solutions found by the two doubly-hybrid meta-heuristic methods are global optimal. Table 3 shows the optimal solutions of the decision variables and expected system cost found by the three algorithms.

It is observed that all the three algorithms lead to the same solutions, which is guaranteed to be the global optimum since the ESA is used as a comparison. The ESA requires more than 30 days to find the optimal solutions (exhaustively), whereas the two doubly-hybrid approaches take a few minutes. This result shows that the accuracy of both the hybrid algorithms is as good as that of the ESA, but the efficiency of the doubly-hybrid algorithms are superior.

Table 3. Optimal solutions of the three algorithms.

Algorithm	*JTEC*(\$/yr)	*n*	*q*	*k*	*L* *(day)*	*s*	CPU/M_CPU (s)	NFE/M_NFE	Suc_ Rate
ESA	3,297.64	123	3	87	56	1.90	3,199,302.72	1,023,814,404	-
PSO+IHS+HJ	3,297.64	123	3	87	56	1.90	432.89	100,131	0.50
MDE'+IHS+HJ	3,297.64	123	3	87	56	1.90	258.80	100,252	0.60

5. CONCLUSIONS

This paper studies the effects of introducing the risk of obsolescence into an integrated lead time controllable consignment stock inventory system with buyer's space limitation and stochastic demand. A four-factor inventory model is developed to jointly determine the optimal value of five decision variables that minimize the annual JTEC of the system. Due to the complexity of the problem structure, analytical solutions are not presented. Instead, two novel doubly-hybrid meta-heuristic algorithms are utilized to find the global optimum of the models. Numerical examples showed that both the doubly-hybrid algorithms perform well both in the sense of the CPU time and the success rate. The results obtained in this study help understand the role of obsolescence, stochastic demand rate, the buyer's space limitation, controllable lead time, and the CS mechanism. Moreover, the successful use of doubly-hybrid meta-heuristic algorithms to inventory problems provides a possible way of solving more difficult and complicated models in the future.

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