Procurement Policy of Vulnerable Parts with Jointly Distributed Lifespan

Cun Rong Li [1], Bhaba R. Sarker [2*], Hui Zhi Yi[2], and Meng Yu[1]

[1]School of Mechanical & Electronics Engineering
Wuhan University of Technology
Wuhan, Hubei 430070, P. R. China

[2]Department of Mechanical & Industrial Engineering
Louisiana State University
Baton Rouge, LA 70803, USA

ABSTRACT

Vulnerable parts exist in most machines or equipment. Failure of those vulnerable parts may result in damage of the machine or the product, and may sometimes cause unnecessary down-time within a production cycle, all of which increases the production costs. Therefore, such failures should be minimized and the parts should be replaced at a point before the end of their expected lifespan, which usually follow some kind of distributions and can be known due to its mass usage attributes. On the other hand, too early replacement of the parts also means increased maintenance costs, increased inventory, and increased parts purchasing cost. This study developed a mathematical model to deal with the issue of determining the optimal stopping time of vulnerable tools. Under the preemption situation, a periodic procurement policy is investigated simultaneously to find the optimal order cycle and eventually minimize the total cost. A case study is given to show the feasibility of the model and several parameters are analyzed to show the impact on the total cost and decision variables.

1. INTRODUCTION

Lifespan estimation has been studied in many areas especially for mechanical components in a manufacturing company. Vulnerable parts of a machine or equipment are not constantly purchased in a short notice or request. When the vulnerable parts lose effectiveness, it sometimes causes serious consequence. So it is important to dig into this situation and try to make right decision on how to choose the right stopping point of part lifespan. Vulnerable parts in a machine or equipment are usually changed with scheduled timetable. Under the situation of different stopping time, optimal order size and order cycle of the vulnerable parts also have great significance in maximizing the profitability and minimizing the costs. The goal of this paper is to propose a cost minimization model (CMM) considering the jointly distributed lifespan of vulnerable parts which have similar function. The economical maximum allowable usage time of the vulnerable part and optimal ordering cycle time can be resolved by applying the CMM model by minimizing the total cost.

There are many researches that concentrated on the lifespan distribution of a product, mechanical part or assembly. Hiromichi *et al.* (2010) analyzed the lifetime distribution of more than 0.7 million products sold in Japanese supermarkets in the area of lifespan estimation. Using Weibull and the artificial neural networks approach, Mazhar *et al.* (2007) built functional relationships between historical data and the parameters of the distribution of parts' remaining lifespan, which made it possible to capture and interpret the existing complex stochastic distribution of the remaining lifespan of components so that we can make best use of it. Li and Sarker (2013) investigated the traditional metal cutting theory to predict the cutting tool lifespan efficiently based on the *F.W.Taylor's* equation using fixed cutting factors. Jun *et al.* (2008) studied an evaluation of the durability of vehicle parts for service life and safety in the early stages of design, then they used the dynamic stress time history and the modal stress recovery method to predict the lifespan and applied to the design of vehicle parts. As to the facet of cost assessment related to product lifespan, Qian and Ben-Arich (2008) presented a methodology for integrating parametric cost estimation with activity-based costing which can be applied to cost estimations of the design of machined parts in one manufacturing factory. From the aspect of statistics, many researches put joint distribution into use of operational research (Laih, 2013, Conti *et al.*, 2013). Some contributions came from reliability engineering, like, used for joint redundancy and imperfect preventive maintenance optimization with series–parallel multi-state systems (Levitin and Lisnianski, 1999). However, there is rarely detailed discussion on influence of parts lifespan with procurement policy.

According to specific influence of the vulnerable parts on the machine, a dependent distribution of the machine needs to be discussed. Former literature can be hardly found on the combination of joint distribution for vulnerable

* Corresponding author: Tel.: (1) 225 578 5370; E-mail: *bsarker@lsu.edu*

parts and its host machine. Further, based on the dependent distribution of the machine, the expected effective working time of the machine need to be discussed which impose an important impact on the procurement policy of the vulnerable parts. So this paper is to construct a procurement policy model on the vulnerable parts based on their predetermined distributions.

2. FORMULATION FOR OPTIMAL COST ESTIMATION

The objective of this section is to formulate the cost estimation model with which the total cost of purchasing, holding, penalty can be minimized.

2.1. ASSUMPTIONS AND NOTATIONS

The combined issue of preemption and procurement for vulnerable parts is quite complex. In order to make a discreet discussion in this paper, several assumptions are taken into account during the formulation of the cost minimization process.

Assumptions:

In order to model the issue in a relatively simple way, the following assumptions are necessary in this study:

1. If one vulnerable part fails, the machine encounters a downtime due to the key function of the part.
2. The demand of the product is continuous, and the machine works under a mass production system with insignificant variation of working time.
3. All parts of the machine are independent so the failure of one part will not lead to the failure of the others.
4. There are multiple vulnerable parts of the same type in a single machine.
5. Failure of a vulnerable part damages the work piece, thus results in a penalty cost.

Notations and definitions summary:

All definitions of parameters, variables, throughout the paper are listed as below:

c_f : Fixed cost for every order ($/order),

c_h : Unit holding cost of a part ($/part/ year),

c_u : Unit purchase (variable) cost of a part ($/part),

c_p : Unit penalty cost of part failure ($/part),

C_f : Fixed cost of order Replenishment per year ($),

C_h : Annual holding cost ($),

C_{pc} : Annual purchasing cost of parts ($),

C_{pt} : Annual penalty cost ($),

D_p : Demand of vulnerable parts required per year,

Q : Order size of parts (parts/order),

T : Lifespan of vulnerable part (in time-unit),

T_c : Cycle length, the length of time between placement of replenishment orders (year),

T_M : Working time for machine or equipment (in time-unit),

T_w : Total working time for a single machine per year (or equipment) (time-units/year),

$T_M^{\max}$: Maximum allowable working time for machine or equipment (in time-unit),

TC : Total cost ($/year).

2.2. PROBLEM DEFINITION

Equipment or machine is a key factor in regular manufacturing process in manufacturing industry. The machine is an assembly which consists of many functional subassembly or parts, and there usually exists several vulnerable parts which play an important role in machining process. Under this situation, it is assumed that the lifespan of the vulnerable part follows a general distribution: $T \sim f(T)$. If one of the parts fails, there is a need for change of this part, and will also result in damage penalty, c_p. On the contrary, if the parts with high probability of potential failure are changed before any of the parts fail, no penalty occurs. However, the preemption of the vulnerable parts results in more replenishment for changing of the parts, and increase the purchasing cost and holding cost accordingly. So this is a balance issue for preemption and inventory control with the vulnerable part.

In this issue, every part has a purchasing cost of c_u. Unit holding cost per year is given by c_h, there exists a fixed purchasing expense, c_f for every order. Based on the optimal decision of the preemption of the machine (or equipment), $T_M^{\max}$, with a rational order cycle, T_c, as decision variables, for a designated period (like, a year), minimizing the total cost, TC is the objective of this paper.

2.3. ESTIMATION OF EXPECTED LIFESPAN FOR A MACHINE WITH VULNERABLE PARTS

The vulnerable parts are assumed to be the same type of part, they vary from each other due to their individual minor difference and usually follow a generic distribution $T \sim f(T)$. Under this circumstance, the maximum working length of the machine depends on the shortest lifespan of all the vulnerable parts. The shortest lifespan of all the parts is the bottleneck, and it follows the well-known *cask theory*. Here, T_i, for $i = 1,2,...n$ represents the practical lifespan of each parts in the machine. The maximum working length for the machine, T_M, can be easily expressed as

$$T_M = \min(T_i) \text{, for } i = 1,2,...n . \tag{1}$$

Since each part follows the same distribution $T \sim f(T)$, and there are n vulnerable parts in the specific machine, according to Equation (1), the distribution of the machine follows a distribution jointly depend on all the parts. The distribution of the maximum working time for a machine is defined as $g(T_M)$, where T_M is the minimum lifespan of all the vulnerable parts. The cumulative probability function for $g(T_M)$ is defined as $G(T_M)$. Since each part of the machine are independent variables, the distribution of $g(T_M)$ can be deducted with *set theory*.

Since $T_M = \min(T_i)$, for $i = 1,2,...n$, the cumulative probability, $G(T_M^{\max})$, can be given as $G(T_M^{\max}) = P\{\min(T_i) < T_M^{\max}\}$. By deduction, the final expression of $G(T_M^{\max})$ illustrated in Figure 3 is described as

$$G(T_M^{\max}) = 1 - [1 - F(T_M^{\max})]^n . \tag{2}$$

By differentiation on equation (2), the probability density function of maximum working time for a machine is given by

$$g(T) = n[1 - F(T)]^{n-1} f(T) . \tag{3}$$

Under the distribution described in equation (3), if a machine is assumed to be the stopping time for parts changing at the time $T = T_M^{\max}$, there are some probability of machine failure before the stopping point, $T_M^{\max}$, and also probability of regular running at the stopping time point. As a result, the expected effective working time for a machine composed of two parts which is marked as *area A* and *B* in Figure 1.

For the portion with $T < T_M^{\max}$, marked area, A, as shown in Figure 1, the expected effective working time in the range $T < T_M^{\max}$, can be expressed as

$$E(T)\Big|_{T=T_M^{\max}} = \int_0^{T_M^{\max}} Tg(T)dT . \tag{4}$$

Replace $g(T)$ with equation (3) in equation (4), the effective expected working time in area A is given by

$$E(T)\Big|_{T=T_M^{\max}} = \int_0^{T_M^{\max}} T\{n[1 - F(T)]^{n-1} f(T)\}dT \tag{5}$$

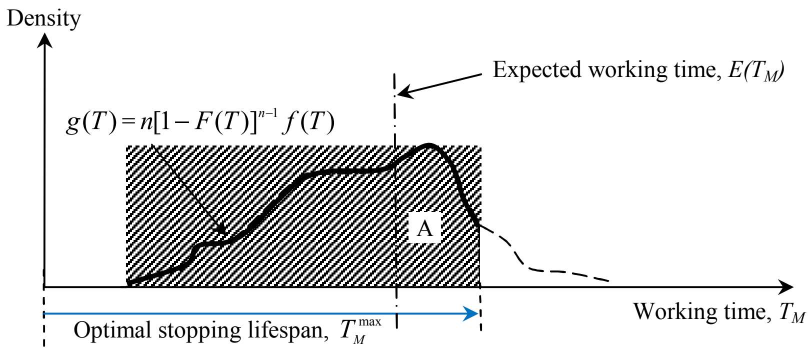

Figure 1. Expected effective working time for a machine.

For the other portion marked as area B in Figure 3, the effective working length of the machine exceeds the maximum allowable usage time, $T > T_M^{\max}$; the practical working length is stopped at the time T_M . So, the expected time length for this preempted portion can be *defined* as

$$E(T_M^{\max}) = T_M^{\max} \int_{T_M^{\max}}^{\infty} g(T)dT = T_M^{\max}\left[1 - G(T_M)\right], \tag{6}$$

where $g(T)$ follows a distribution described in equation (3).

With the portion marked with *B* area, combine equation (6) with equation (2), the expected value of this area can be given by

$$E(T_M^{\max}) = T_M^{\max}[1 - F(T_M^{\max})]^n . \tag{7}$$

Since the total population of the machine is comprised of two groups, *A* and *B* as explained above, each group of the probability has different expected working length for that machine. Under this situation, we now define the total expected working length of both groups with an assumption of $T_M < T_M^{\max}$ as

$$E[g(T)]_{A+B} = E(T)\big|_{T=T_M^{\max}} + E(T_M^{\max}) . \tag{8}$$

2.4. MODELING FOR MINIMIZATION OF TOTAL COST

Under an assumption of situation for massive production, the annual working time for a machine is fixed. As a result, based on an approximate regular continuous working length (which has an expected time length expressed in equation (8) once the maximum stopping time, $T_M^{\max}$, is determined) between the vulnerable parts changing, the vulnerable parts is assumed to follow a fixed-period, T_c, review policy where an order of Q parts/cycle is placed to meet the total yearly demand of vulnerable parts. The total cost includes fixed cost of a replenishment order C_f , holding cost C_h , purchasing cost C_{pc} and penalty cost C_{pt} . Then the total cost for a purchasing cycle, TC can be expressed as

$$TC = C_f + C_h + C_{pc} + C_{pt} . \tag{9}$$

The total working time for a single machine in a year is T_w . From equation (8), the expected continuous working length without parts changing is $E[g(T)]_{A+B}$. The number of vulnerable parts for changing every time is n because there are totally n same parts in a machine. So, the total demand of vulnerable parts is

$$D_p = \frac{nT_w}{E[g(T)]_{A+B}} \tag{10}$$

T_c is defined to be the purchasing cycle (in year), and c_f is the fixed purchasing expense for every time, so the annual total fixed purchasing cost can be calculated as

$$C_f = \frac{c_f}{T_c}, \tag{11}$$

Annual holding cost, C_h, can be calculated as the unit holding cost c_h multiplied by the average inventory, which can be expressed as $I = Q/2 = D_p T_c / 2$. Replacing D_p with equation (10), the annual total holding cost is given by

$$C_h = \frac{nT_w c_h T_c}{2E[g(T)]_{A+B}} \tag{12}$$

For annual parts purchasing cost, it is only an issue with purchasing quantity, so, it can be easily expressed as

$$C_{pc} = \frac{nT_w c_u}{E[g(T)]_{A+B}} \tag{13}$$

Annual penalty cost is related to the yearly total failures of the machine (or equipment). $G(T_M^{\max})$, the cumulative probability of the machine failure that occurs before the stopping time, $T_M^{\max}$, is expressed in equation (2) as $G(T_M^{\max})$ $=1-[1-F(T_M^{\max})]^n$. The annual total penalty cost, $C_{pt} = c_p D_p G(T_M^{\max})$, combined with equation (10), thus can be rewritten as

$$C_{pt} = \frac{nT_w c_p\{1-[1-F(T_M^{\max})]^n\}}{E[g(T)]_{A+B}}. \tag{14}$$

The total cost, TC, by merging equation (9), (11), (12), (13) and (14), can be deducted as

$$TC(T_M^{\max}, T_c) = \frac{c_f}{T_c} + \frac{nT_w c_h T_c / 2 + nT_w c_u + nT_w c_p\{1-[1-F(T_M^{\max})]^n\}}{E[g(T)]_{A+B}}. \tag{15}$$

As shown in equation (15), there are mainly two components, and one component (fixed purchasing cost) is a function of order cycle, T_c. The other one regarding to holding cost, part purchasing cost and penalty cost is related to the expected maximum working length, $E[g(T)]_{A+B}$. These relationships make the analysis more observable.

3. Solution for CMM Function

There are two decision variables in equation (15), and the objective of this function is to minimize the total cost. In order to find the stationary point of this equation, differentiation method can be employed to resolve this issue.

3.1. Mathematical Solution

In order to find the stationary points for the objective function expressed in equation (15), partial differential methods on both T_c and $T_M^{\max}$ are required to retrieve its close form expression. By differentiation on T_c, $\partial TC(T_M^{\max}, T_c)/\partial T_c = 0$ yields

$$T_c = \sqrt{\frac{2c_f\left(\int_0^{T_M^{\max}} T\{n[1-F(T)]^{n-1} f(T)\}dT + T_M^{\max}[1-F(T_M^{\max})]^n\right)}{nc_h T_w}} \tag{16}$$

By differentiation on $T_M^{\max}$ with objective function (15), let $\partial TC(T_M^{\max}, T_c)/\partial T_M^{\max} = 0$, a final expression is given as follows (See Appendix B for detailed deduction process).

$$nc_p f(T_M^{\max})\left(\int_0^{T_M^{\max}} T\{n[1-F(T)]^{n-1} f(T)\}dT + T_M^{\max}[1-F(T_M^{\max})]^n\right)$$
$$-\left(c_h T_c / 2 + c_u + c_p\{1-[1-F(T_M^{\max})]^n\}\right)[1-F(T_M^{\max})] = 0 \tag{17}$$

In order to eliminate one of the decision variables, by merging equation (16) and (17), a final expression with only one decision variable, $T_M^{\max}$, is given by

$$\left(nc_p f(T_M^{\max}) - \frac{c_f}{nT_w}[1-F(T_M^{\max})]\right)\left(\int_0^{T_M^{\max}} T\{n[1-F(T)]^{n-1} f(T)\}dT + T_M^{\max}[1-F(T_M^{\max})]^n\right)$$

$$-\left(c_u + c_p\{1-[1-F(T_M^{\max})]^n\}\right)[1-F(T_M^{\max})] = 0. \quad (18)$$

From equation (18), we cannot get a close form expression for the decision variable, $T_M^{\max}$. But using equation (15), the computational amount can be decrease dramatically, so a minimum-oriented searching (MOS) algorithm is employed to resolve its approximate solution. With further calculation, decision variable, T_c, the order size, Q, which depends on T_c, and minimum total cost, TC, can be resolved accordingly.

3.2. MINIMUM-ORIENTED SEARCHING ALGORITHM

Given every value of $T_M^{\max}$, a corresponding result, T_c, can be computed with equation (16), and then the minimum value of $TC(T_M^{\max}, T_c)$ is determined in this searching process. The simplified MOS algorithm can be illustrated as a flow chart shown in Figure 2.

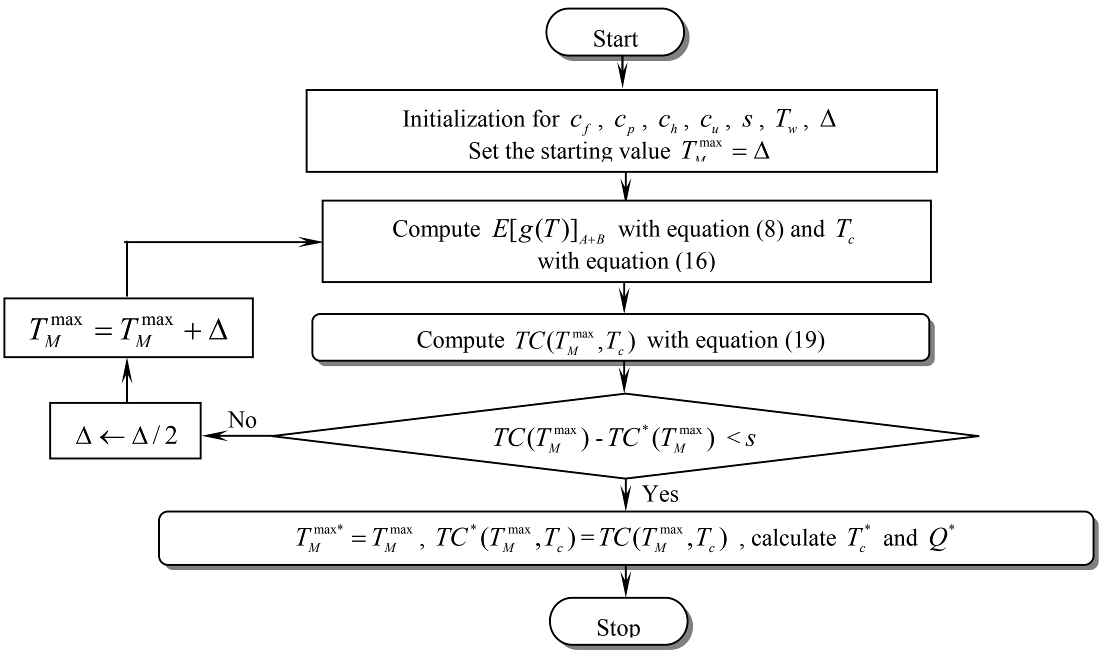

Figure 2. Simplified flow chart of MOS searching algorithm.

3.3. A CASE STUDY

In order to verify the CMM model by employing MOS searching Algorithm, a case study is conducted to show its feasibility. In this example, normally distributed vulnerable parts are given to pursue this exploration. Experimental equipment is designed for testing of electromagnetic valve. The equipment has 4 customized gear-racks which are key functional parts for testing process, and the part is also a quick-wear part due to its frequent changing of testing work piece. Under this situation, the lifespan of these vulnerable parts follows a normal distribution with μ_p = 200 hours and σ = 5. An equipment need to work for a total 3650 hours per year under a massive production requirement which means a steady demand of working time. For parts purchasing process, there is a fixed order cost, c_f = \$30/order and unit cost, c_u =\$20/unit. In testing process, unexpected failure for each of the 4 parts has potential damage for the testing work piece and result in an average penalty cost, c_p =\$1/unit, consequently, all the 4 parts need to be changed to avoid further damage. For every part in the stock, there is an annually holding cost, c_h =\$3/unit/year. Under this situation, a maximum working length (or stopping time) for the equipment, $T_M^{\max}$, and order cycle, T_c are to be resolved to minimize the total cost, TC.

By running the MOS program with an initial step size Δ=10 and stopping criteria s = \$0.01, Table 1 shows the searching result of this vulnerable parts changing and procurement problem. The optimal decision variables, $T_M^{\max}$=191.0006 hours, order cycle T_c=0.382 years $\approx$ 139 days, order size Q=13.09 $\approx$ 13 units and the minimized total cost TC^* =\$2,918.547.

Table 1 show that the MOS algorithm is efficient in finding the optimal solution under designated stopping criteria, S . The convergence of this algorithm is proved to be robust in similar problem simulation.

Table 1. Searching result for CMM with MOS algorithm.

Step No.	Stopping time, T_M^{max}	Stop criteria, S	Step size Δ	Total cost, TC
1	11.0000	474,238.335	10.00000	47,924.996
2	21.0000	22,691.014	10.00000	25,233.982
3	31.0000	8,071.243	10.00000	17,162.739
…	…	…	..	…
16	191.1563	1.926	0.15625	2,920.467
17	191.0781	0.910	0.07813	2,919.451
18	191.0391	0.442	0.03906	2,918.983
…	…	…	…	…
22	191.0024	0.027	0.00244	2,918.568
23	191.0012	0.013	0.00122	2,918.554
24	191.0006	0.007	0.00061	2,918.547

4. Parametric Analysis

The maximum working length of a machine, T_M^{max} , is determined by the minimum lifespan of all the vulnerable parts in this machine, so the number of parts, n , has an impact on regular working time. Besides, from the view of management, holding cost, c_h , and penalty cost, c_p also impose significant influence on the order size and machine stopping time, and eventually have impact on the total cost. TC . In this sector, insight analysis of c_h , c_p and the parts number, n is conducted as influence on the total cost.

4.1. Variation of Penalty Cost c_p and Holding Cost c_h

Penalty cost is a factor which influence the total cost when failure occurs, as a result, in order to decrease the negative impact of penalty cost, an earlier preemption of the machine for parts changing is preferred. For verification, a set of penalty cost, c_p and holding cost, c_h are tested to compare the difference of optimal value of T_m . By adopting the same example in Section 3.3, testing parameters, c_p is set to 0.5, 1.0, 2.0 dollars per unit, and c_h is set to 3.00, 4.00, 5.00 dollars per unit per year, respectively. The computational results with MOS algorithm are shown in Table 2.

Table 2. Comparison of impact on T_m for different values of c_p and c_h .

Holding cost c_h (\$/unit)	Penalty cost c_p (\$/unit)	Stopping time T_M^{max*} (Hours)	Order cycle T_c^* (Days)	Order size Q^*	TC^* (\$)
3.00	0.50	191.0012	139	13	2,909.21
3.00	1.00	191.0006	139	13	2,918.55
3.00	2.00	191.0003	139	13	2,937.22
2.00	1.00	191.0006	171	16	2,889.72
4.00	1.00	191.0006	121	11	2,942.85

Table 2 also shows that increase of penalty cost, c_p , resulting in earlier preemption for the working machine and it is beneficial to the final result of total cost. Decease of order size is preferred when higher unit holding cost rises. However, total cost get higher whenever what policy is adopted. In management aspect, reasonable management level with rational value of c_p and c_h can thoroughly decrease the total cost to a relative low level.

4.2. Influences of Variant parts, n

Equation (3) gives the probability density function of failure time for n parts, where n , represents the quantity of the same vulnerable parts in a machine and have impact on the maximum allowable working time for the machine. It is depicted that with the increase of the vulnerable parts in a machine, the expectation of the function, $g(T)$, decreases. However, the variation of the function falls into a smaller range. Since the machine is affected by n parts, the allowable maximum working time decreases accordingly.

In order to reveal the influence of the quantity of the vulnerable parts, different quantity of parts (3, 4, 5, and 6) are input into CMM model by computing with MOS algorithm. Table 3 shows the comparison of the impact of TC^* with different quantity n. With the increase of the quantity of vulnerable parts, the maximum allowable stopping time, $T_M^{\max *}$ decreases. The change with $T_M^{\max *}$ also reflects the decrease of a jointly overall expected working time with the vulnerable parts.

Table 3. Comparison of impact on TC^* with different quantity n.

quantity n **(units)**	**Stopping time** $T_M^{\max *}$ **(Hours)**	**Order cycle** T_c^* **(Days)**	**Order size** Q^*	TC^* **($)**
3	191.0195	161	15	2,196.12
4	191.0006	139	13	2,918.55
5	191.0003	124	12	3,649.37
6	191.0001	113	11	4,391.26

5. CONCLUSION

This paper studied the procurement policy for vulnerable parts which are key components in a machine. The dependent distribution of the machine, which is jointly formulated from the generic distribution of the vulnerable parts, is investigated. Under a preemption of machine working time, the expectation of the transformed distribution of the machine is studied. Based on this research, a procurement CMM model was proposed to minimize the total cost. In order to verify the feasibility of the CMM model, an example with normally distributed vulnerable parts was given to simulate the computational process and the result illustrated the effectiveness of the model. Further, several parameters are studied to analyze to what extent these parameters influence the final total cost.

ACKNOWLEDGEMENTS

This research work at Louisiana State University, Baton Rouge, LA 70803 was supported by *China Scholarship Council* under the Grant Number: 2011-69-5508.

REFERENCES

[1] *Conti, P. L., Marella, D., Scanu, M., (2013), "Uncertainty analysis for statistical matching of ordered categorical variables,"* Computational Statistics & Data Analysis*, **68**(12): 311–325.*

[2] *Hiromichi, U., Tsutomu, W., Misako, T., (2010), "Life-span distributions of supermarket products,"* Journal of Physics*, 221(2010)012018.*

[3] *Jun, K. J., Park, T. W., Lee, S. H., Jung, S. P., Yoon, J. W., (2008), "Prediction of fatigue life and estimation of its reliability on the parts of an are suspension system,"* International Journal of Automotive Technology*, DOI 10.1007/s12239-008-0088-4.*

[4] *Laih, Y. W., (2013), "Measuring rank correlation coefficients between financial time series: A GARCH-copula based sequence alignment algorithm,"* European Journal of Operational Research*, **232**(2): 375–382.*

[5] *Levitin, G,. Lisnianski, A., (1999), "Joint redundancy and maintenance optimization for multistate series-parallel systems,"* Reliability Engineering & System Safety*, **64**(1): 33–42.*

[6] *Li, C. R., Sarker, B. R., (2013), "Lifespan prediction of cutting tools for high-value-added products,"* International Journal of Advanced Manufacturing Technology*, **69**(5-8): 1887–1894.*

[7] *Mazhar, M. I., Kara, S., Kaebernick, H., (2007), "Remaining life estimation of used components in consumer products: Life cycle data analysis by Weibull and artificial neural networks,"* Journal of Operations Management*, **25**(2007) 1184–1193.*

[8] *Qian, L., Ben-Arieh, D., (2008), "Parametric cost estimation based on activity-based costing: A case study for design and development of rotational parts,"* Int. J. Production Economics*, **113** (2008): 805–818.*