Batch Size Optimization Based on Production Part Cost

Christina Windmark*, Kathrine Spang, Fredrik Schultheiss, and Jan-Eric Ståhl

Division of Production and Materials Engineering
Lund University
Lund, 223 63, Sweden

Abstract

When investigating different location and/or system designs the possible variables to take into consideration can differs between the alternatives. Different production system will have different optimal working conditions and hence should be compared with parameters suitable for the actual production system. When planning production and calculating production costs the batch size is of high interest. Based on a manufacturing part cost model, this paper will present a new model, close connected to the production system, integrating production performance, set-up times, material costs, material handling costs and tied capital, giving the production economic optimal batch size. The aim is to give companies a model for determining the economic optimal batch size in order to use this knowledge to make strategic decisions regarding production planning. Mathematical simulations are performed to analyse the differences in result from the developed model and Wilsons existing standard method for calculating the economic order quantity, hence to verify the importance of making an in-depth analysis, taking the production system into consideration. The advantage of the developed model is the usage of production costs based on variable batch sizes, giving a more accurate outcome.

1. Introduction

Analysis of production costs is generally an essential part while evaluating products, production systems and production location. The costs of producing one product depend on different parameters as quality, cycle time, lead time and batch size. When planning and executing manufacturing operations the batch size can in many cases be one of the most important parameters to consider. When having small batch sizes the time for set-up has to be divided over a small number of products, which could make the costs considerably higher than when producing large batches. On the other side having large a batch size the set-up costs can be low per product but storage costs, costs for tied capital and costs for product obsolescence considerably will be higher. Besides the most obvious difference in cost connected to batch size the downtime rate, quality rate and material losses could be connected to the batch size, especially if the production lines/machines have long run-in time and require a lot of adjustments at the beginning of production. The costs connected to these scenarios have to be taken into consideration to attain a comprehensive analysis regarding batch size. In the case of strictly customer order driven manufacturing, the batch size is determined mainly by the level of yearly demand and the times at which orders are received and delivery takes place. When many of the orders during a given year are of contractual character, one may well decide to combine several batches and, in connection to this, manufacture products that are to be stocked.

This paper will present a model for analysing the economic optimal batch size based on manufacturing and inventory costs. The model will be compared with the model of economic order quantity (EOQ) first presented by Ford [1] and later on Hadley and Whitin [2]. The presented model have parameters depending on batch size, which makes it possible to generate the economic optimal batch size without using fixed production costs, in contrast to Fords model also known as Wilson's model. Both models will be mathematical simulated and the results compared.

2. Background

There are a numerous number of literature works where the importance of production volume and batch size are raised. When searching for the wording "economic order quantity" on scholar.google.com and www.scopus.com over 1 300 000 respectively 2 400 hits on articles and books are obtained, showing the scale of the field. The optimal level of production order quantity depends on many aspects such as time, economic and synergy effects of producing different products after each other, making set-ups and tool changes more efficient and therefore makes smaller or

* Corresponding author: Tel.: (46) 46-222 43 88 Fax: (46) 46-222 85 04; E-mail: christina.windmark@iprod.lth.se

larger batches dependent on order from costumer. Models for estimating the economic order quantity have been used by a number of different authors, with different views and perspective and different level of generalizations and simplifications. In this paper the authors take the economic aspect into consideration. The model presented in this paper is based on the model developed by Ståhl [3]. In contrast to [3], this paper present the optimal batch size with the total production part cost making the result more visually for the user, when deciding upon batch size. Table 1 presents an extract of the published models within batch size optimization and the parameters used by the authors.

Table 1. Parameters used in batch size optimaztion models.

Authors	**Set-up costs**	**Order cost/shipment costs**	**Stockholding cost vendor**	**Stockholding costs buyer**	**Production rate**	**Demand**	**Batch time**	**Number of shipments per batch**	**Size of shipment**	**Total stock in system**	**Total production cost**	**Unit production cost**	**Price per unit**	**Raw material carrying costs**	**Carrying cost finished goods**	**Quantity raw material ordered**	**Batch size**	**Number of shipments**	**Time between shipments**	**Quantity produced between shipments**	**Manufacturing time**	**Cycle time**	**Downtime/downtime rate**	**Average inventory**	**Inventory built up at end of shipment**
Ford (1913) [1], Hadley and Whitin (1963) [2]	x		x		x	x						x					x								
Banerjee (1986) [4]	x	x	x		x	x						x	x				x								
Cheng (1989) [5]	x			x		x					x	x					x								
Golhar and Sarker (1992) [6]	x	x	x		x	x			x					x	x	x	x	x	x	x	x	x	x	x	x
Hill (1999) [7]	x	x	x	x	x	x	x	x	x	x		x					x								
Model presented in this paper	x	x	x		x	x	x		x	x		x		x	x		x				x	x	x		

Most of the models presented in Table 1, with the exception of [5], have in common the simplification that the production cost per part is constant and not dependent on batch size. Cheng [5] present a model where the production cost is dependent on the yearly demand. Ståhl [3] and Persson and Svenle [8] present a model for batch size optimization, where Ståhl [3] present the theoretical framework and Persson and Svenle [8] an extensive case study of the theoretical model on products at a stone crusher manufacturer. The majority of the models in Table 1 do not take account of measures of production losses such as rejection rates q_Q, downtime rates q_S, and the like. An exception is Golhar and Sarker [6], where downtime rate is regarded. Ståhl [3] presents a correlation between batch size and accumulated downtime rate q_S during the production of the batch similar to that as presented in Figure 1, which indicate that the batch size can have a considerable impact on performance parameters. Figure 1 shows five batches where the accumulated downtime rate presented as a function of the batch size and the thick black line represent the trend of the five batches analysed. The function is used when finding the relation between batch size and downtime to use when estimating the optimal batch level based on variable parameters. To get reliable data, the number of batches analysed has to increase considerably compared to the analysis in Figure 1. More studies in the field has to be conducted and it is important to notice that the result from this kind of analysis will be very company specific due to company specific production facilities and different quality assurance levels. This paper will compare the result from the model presented in this paper with the model developed by Ford [1] and have the intention to answer the following questions:

1. How large is the differences between the less elaborate model developed by Ford [1] and a model based on theoretical framework presented by Ståhl [3] regarding batch size and total production part cost?
2. How is the total production part cost affected by the batch size?
3. What are the advantages of the two models?

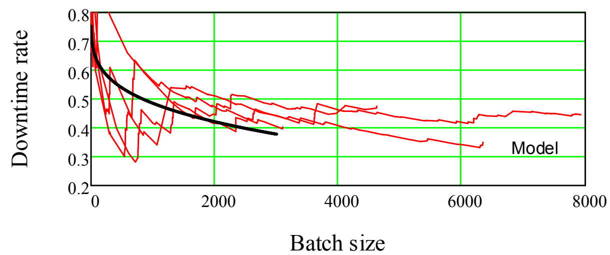

Figure 1. Relationship between downtime rate and batch size for five different batches of one product.

3. THE MODELS

In this section the two models are presented. All parameters used in the equations are specified in Appendix 1. Due to the differences between the two models concerning how different parameters are dependent on the order quantity/batch size different generalisations are made. Table 2 present the assumptions of the models.

Table 2. The simplifications and assumptions of the two models.

Simplifications and assumption common for the two models	
• No products obsoletes in stock • No deficiencies in deliveries occur • Quantity discount from suppliers are not taken into account • The whole batch is processed before entering finished goods inventory or is transported to costumer • There is always a safety stock of 10 % • The set-up time is fixed and non-depending of batch size • Costs of handling incoming orders are constant • Internal transfers are not dependent on batch size • External logistics are not included • Storage holding cost is constant • Material is ordered and received to one batch at a time	
Specific for the model based on Ståhl [3]	Specific for model based on Ford [1]
• Either constant or periodic demand • Equipment hourly cost is not dependent on batch size and market demand and is supposed to be based on the last five years production. • Downtime losses are dependent on batch size	• Production cost per part is constant. • Demand is constant and continues

Figure 2 gives a general of the material flow and production conditions. The production is regarded as one line/machine and there are only two inventories, raw material and finished goods. The model consists of four parts, manufacturing costs, cost of tied capital, costs of inventory and inbound logistic and costs of handling incoming orders.

The model is based on the manufacturing part cost equation presented by Ståhl et al [9], Equation 1, where index M in this case represents manufacturing. In comparison to the original manufacturing part cost model the downtime rate q_S is a function of the batch size N_0. Equation 2 gives an example of such a function. The function is based a mathematical adjustment of measured data and a_1, a_2 and a_3 is variables. When implementing the model this function has to be adopted and adjust to the product investigated.

$$k_M = \left[\frac{k_B}{(1-q_B)(1-q_Q)}\right]_B + \frac{k_{CP}}{MD\cdot 60}\left[\frac{t_0 MD}{(1-q_Q)(1-q_P)}\right]_{CP} + \frac{k_{CS}}{MD\cdot 60}\left[\frac{t_0 MD}{(1-q_Q)(1-q_P)}\cdot\frac{q_S}{(1-q_S(N_0))} + T_{su}\cdot\frac{MD}{N_0}\right]_{CS} + \frac{k_D}{MD\cdot 60}\left[\frac{t_0 MD}{(1-q_Q)(1-q_P)(1-q_S(N_0))} + T_{su}\cdot\frac{MD}{N_0}\right]_D \quad (1)$$

$$q_S(N_0) = a_1 - a_2 \cdot {N_0}^{a_3} \quad (2)$$

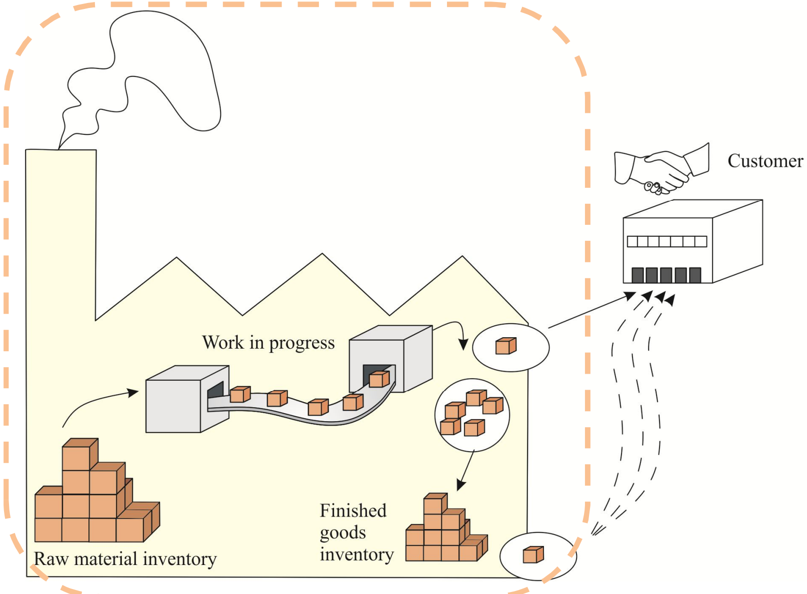

Figure 2. Schematic depiction of the material flow in production, some products go direct to the costumer and other to the finished goods inventory for later shipments to costumer.

Included in the model is cost for tied capital including work in progress, tied capital in raw material inventory and finished goods inventory. Equation 3 to 6 presents the calculations connected to the cost of tied capital where k_{Cap} is the total part cost of tied capital. The cost of tied capital in work in progress is calculated by the mean value between the raw material cost, k_B and the part cost after manufacturing k_M multiplied with the ratio between the time for producing the batch and the total number of minutes in one year multiplied with the interest rate p. In the cost equation of tied capital in raw material inventory, k_{RMI} and finished goods inventory, k_{FGI}, N_C is the customer order quantity direct delivered to the customer and MD the market demand in one year. N_{SS} is the number of part in safety stock. There is always a 10 % safety stock included in these calculations.

$$k_{WIP} = \frac{k_B + k_M}{2} \cdot \frac{p \cdot (t_0 \cdot N_0)}{365 \cdot 24 \cdot 60} \tag{3}$$

$$k_{RMI} = k_B \cdot p \cdot \left(\frac{N_0}{2MD} + \frac{N_{SS}}{N_0}\right) \tag{4}$$

$$k_{FGI} = k_M \cdot p \cdot \left(\frac{(N_0 - N_C)}{2MD} + 0.1\right) \tag{5}$$

$$k_{Cap} = k_{PIA} + k_{RMI} + k_{FGI} \tag{6}$$

The cost equation for inventory buildings and internal transportation is presented in Equation 7, where p_e is the pallet equivalent as presented in [10], k_S the surface area cost/year, A, the total area used for handling and storing products, p_{ptot} the total number of pallet places, t_t the time for transporting the batch and k_G the hurly cost for transportation in-house.

$$k_{GIL} = p_e\left(k_S \frac{A}{p_{ptot}} \cdot \frac{N_0}{2MD} + t_t \cdot \frac{k_G}{60}\right) \tag{7}$$

The costs of new order, k_{NO} is the cost of ordering new parts to the raw material inventory, K_{OP} and the costs connected to receiving and handling new orders, K_{HNO} multiplied with number of batches/year divided with market demand, MD. The cost is presented in Equation 8.

$$k_{NO} = \frac{\frac{MD}{N_0} \cdot (K_{HNO} + K_{OP})}{MD} \tag{8}$$

The total part cost equation is presented in Equation 9 and in Figure 3 examples of the four cost functions and the total part cost function are shown.

$$k(N_0) = k_M + k_{Cap} + k_{GIL} + k_{NO} \quad (9)$$

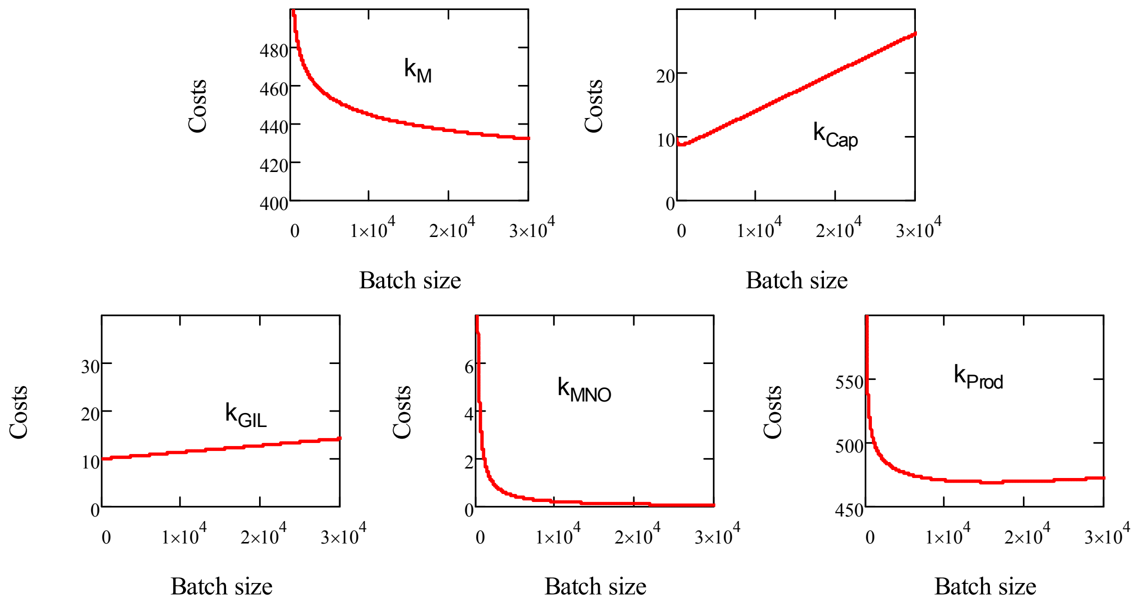

Figure 3. Example of the cost functions included in the model assuming MD = 100 000 and p = 10 %.

The economic order quantity model based on Ford´s model is presented below. Equation 10 presents the ordering cost K_{CO} and Equation 11 the cost of handling and storing a product, h. In equation 12 the economic order quantity, EOQ is calculated using the result from Equation 10 and 11.

$$K_{OC} = \frac{(k_D + k_{CS})}{60} \cdot T_{su} + K_{HNO} + K_{OP} \quad (10)$$

$$h = k_B \cdot p \cdot 1.1 + k_M \cdot p \cdot 1.1 + k_{GIL} \quad (11)$$

$$EOQ = \sqrt{\frac{K_{OC} \cdot MD \cdot 2}{\square}} \quad (12)$$

4. Results

In the following section the result from the mathematical analysis is presented. The models has been analysed using the following values presented in Table 3. An extraction of the results from the analyses is presented in Table 4. Table 4 presents the optimal batch sizes N_{opt} and EOQ and the total part cost based on the batch sizes obtained, for different market demands and interest rates. In the table $k(N_{opt})$ is the total production part cost connected to N_{opt} and k(EOQ) to EOQ. Figure 4 presents four of the simulations of the cost function $k(N_0)$ regarding market demand of 10 000 parts/year and 100 000 parts/year respectively. The optimal batch sizes from the two models are marked with vertical lines and cost increase with 2.5, 5.0, 7.5 and 10 % are marked with horizontal lines. In order to attain a more accurate view of the cost correlation between the batch size and production part cost the raw material cost k_B is subtracted.

Table 3. The values on the paramets used in the simulations. Estimated from experiense, optained in Swedish industry.

Parameter	k_B	k_{CP}	k_{CS}	k_D	q_Q	q_P	q_B	t_0	N_C	T_{su}	p_e	k_a	p_{ptot}	t_t	k_G	K_{HNO}	K_{OP}
Values	350	600	550	350	0.05	0	0.03	3	400	360	0.05	1 400	2 500	50	250	200	2 000

Table 4. The optimal batch size from the two models with market demand (MD) 10 000 to 250 000/year for intrest 1, 5, 10, 20, 30 % and the part cost connected to the batch size.

MD		1 %	5 %	10 %	20 %	30 %
10 000	N_{opt}	7 425	4 410	2 935	1 883	1 435
	EOQ	2 679	1 643	1 232	899	742
	$k(N_{opt})$	473.2	485.5	496.7	513.9	528.2
	k(EOQ)	479.9	492.5	503.1	519.6	533.5
40 000	N_0	23 468	1 2913	8 528	5 278	3 928
	EOQ	5 358	3 287	2 464	1 799	1 485
	k_{Prod1}	507.7	494.2	478.3	468.1	453.8
	k_{Prod2}	476.3	488.6	498.7	514.5	527.9
100 000	N_0	48 375	24 675	15 825	9 568	7 028
	EOQ	8 472	5 196	3 895	2 844	2 348
	k_{Prod1}	448.0	459.4	469.3	484.7	497.7
	k_{Prod2}	475.6	487.8	497.9	513.4	526.8
250 000	N_0	92 625	41 255	26 375	14 935	10 840
	EOQ	13 400	8 216	6 159	4 496	3 712
	k_{Prod1}	441.1	453.1	463.0	478.2	490.9
	k_{Prod2}	475.3	487.5	497.5	513.0	526.3

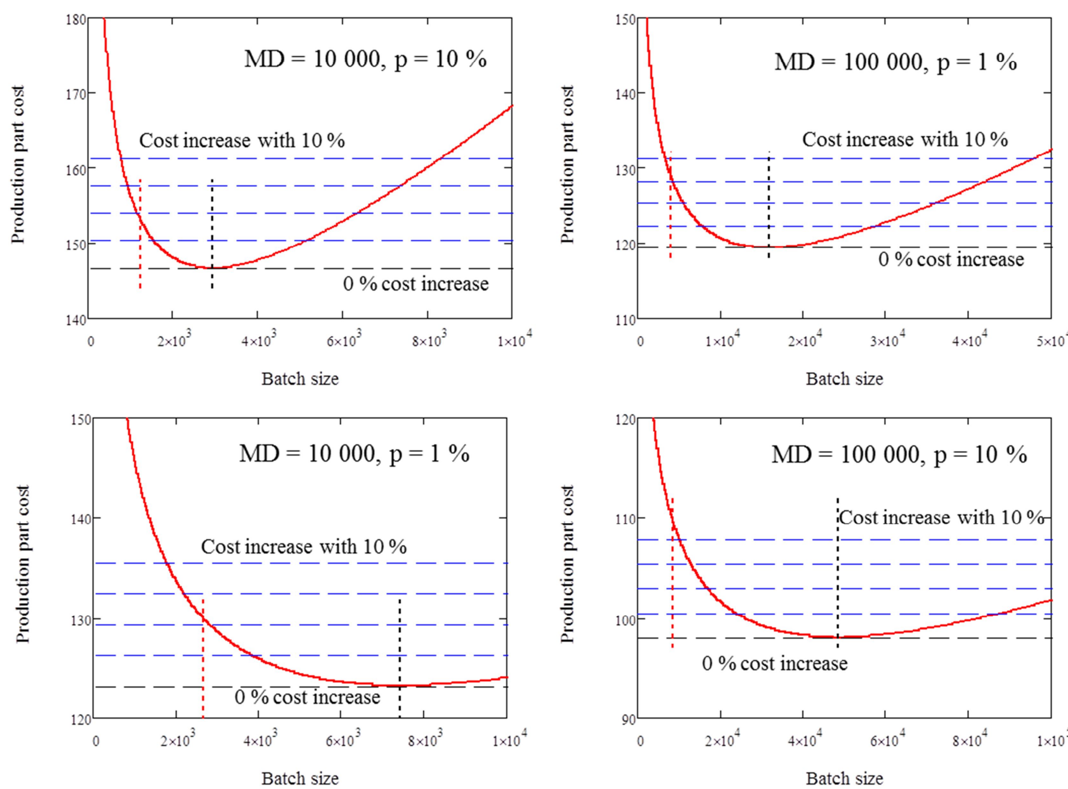

Figure 4. To the left, market demand 10 000, to the right market demand 100 000, the diagrams in the top have intrest rate 10 % and in the bottom interset rate 1 %. The function is k (N_0). The marking to the left is optimal batchsize according to Fords model and the right according to the presented model in this paper and also the minimum value of the function k(N_0).

5. DISCUSSIONS AND CONCLUSIONS

The results presented in Figure 4 indicate that a rather large span of batch sizes gives an economic optimal production cost, allowing a company to use a large variety of batch sizes without compromising costs. When studying Figure 4 it is clear that the derivate of the cost function is larger for smaller batch sizes than for larger. This can also be seen in results from Ford [1]. The result indicates that the economic risk in both models is higher when producing to small batches than to large. In [11] Liker describes the benefits of lowering buffers and stock level, hence decreasing batch sizes in order to see problems in the production, symbolically called the Japanese sea. When a company is active in production development and continually works with improvements but cannot in a near future solve problems concerning production rate and performance, a higher batch size can be advantageous. Especially if the production cost is not considerably higher. However the choice of having larger batch sizes in order to deliver products on time to costumers requires constant update and transparency by varying the batch size to see the actual performance of the production system. This is illustrated by variation in sea level to both get a transparent production where continues improvements can be made and at the same time make the company more profitable and competitive by delivering products on time. The motive in producing parts that are to be kept in stock should not be to avoid the high costs of start-up times and set-up times this can involve, but rather to improve manufacturing technology and profitability.

This paper shows the difference in result using the basic formula of Ford [1] and the more in-depth model based on previously presented by Ståhl [3]. The model based on Ståhl [3] gives a much higher economic order quantity/batch size than the model developed by Ford [1]. However the cost difference is not near the relative differences in batch size. Other authors have already mentioned the low impact of the changes in batch size on the part cost, for example [12]. The economic batch size could be a large span of number of parts. Yet the differences in cost is not negligible between the models, especially not when larger market demand is involved. Figure 4 shows that the mathematical simulation performed in this study gives a cost increase with about 10 % as compared to when using Ford's model instead of the model presented, assuming that market demand is 100 000 parts/year. Important to note is that this result is valid only for the values simulated.

The main advantage of the model developed by Ford is the simplicity of the model and according to the result presented in this paper the economic outcome is close to optimal for smaller market demands. The benefits of using the model presented is the possibility to use production cost dependent on batch size and to easy illustrate the part cost together with the batch size. A problem occurring using the model based on Ford is to know the value of manufacturing cost per part, which is dependent on batch size. It is common that the only manufacturing costs connected to batch size is the set-up costs and the current model makes it possible to take other cost correlations into account. The essential application area of the model is the possibility to have manufacturing cost dependent on batch size when estimating the optimal batch size. The model presented in this paper could be useful when pricing, making the company aware of the part cost for a certain order quantity ordered by the costumer.

Further research should be intended to investigate further correlations between the batch size and performance parameters, company studies have shown that there could be a strong correlation with start of production and quality losses. More activities as external logistics and batch size dependent cost in internal logistics should be added to the model investigated to give better analyses.

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APPENDIX 1: LIST OF SYMBOLS

Parameter	Description	Value
A	Area used when handling and storing products	m^2
a_1, a_2, a_3	Constants	-
EOQ	Economic order quantity (Wilson´s formula)	unit
h	Handling and inventory costs per part	currency/unit
k_M	Manufacturing costs per part	currency/unit
k_B	Material costs per part	currency/unit
k_{Cap}	Costs of tied capital per part	currency/unit
k_{CP}	Hourly machine costs during production	currency/h
k_{CS}	Hourly machine costs during downtimes and adjustments	currency/h
k_D	Salary costs	currency/h
k_{FGI}	Tied capital in finished goods inventory per part	currency/unit
K_{HNO}	Costs of handling a new order in production	currency/batch
k_{GIL}	Costs per part of inbound logistics	currency/unit
k_{NO}	Costs of new orders	currency/unit
K_{OC}	Ordering costs	currency/batch
K_{OP}	Costs for order processing	min
k_{Prod}	Total production part costs	currency/unit
k_{RMI}	Tied capital in raw material inventory per part	currency/unit
k_S	Costs of surface area	currency/m^2
k_{WIP}	Tied capital in work in progress per part	currency/unit
MD	Market demand	unit/year
N_0	Nominal batch size	unit
N_C	Customer order quantity	unit/order
N_{opt}	Optimal batch size	unit
N_{SS}	Number of parts in safety stock	unit
p	Interest rate	-
p_e	Pallet equivalent	pallets/unit
p_{ptot}	Total number of pallet places	number
q_P	Relative rate reduction	-
q_Q	Rejection rate	-
q_S	Downtime proportion	-
t_0	Nominal cycle time per part (for line production the through-put time)	min
t_t	Time for transportation	min
T_{su}	Set-up time	min