H_∞ Fault Detection Filter Design for Networked Control Systems in the Continuous-time Domain

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ABSTRACT

This paper deals with the problem of fault detection filter design for a class of networked control systems. Under the assumptions of network-induced time delay being unknown but bounded, packet dropouts and packets out of sequence being unavoidable, a system model for networked control system is firstly introduced in the continuous-time domain. Then an observer-based H_∞ fault detection is formulated and, by applying the Lyapunov-Krasovskii functional approach, a delay-dependent sufficient condition on the existence of the H_∞ fault detection filter (FDF) is derived in terms of matrix inequality. Furthermore, an algorithm is proposed to get a feasible solution to the H_∞ fault detection filter gain matrices in terms of linear matrix inequalities (LMIs) using a cone complementary technology. A simulation example is given to demonstrate the effectiveness of the proposed method.

1. INTRODUCTION

Over the past decades, fault detection and isolation (FDI) was an active field of research due to an increasing demand for higher performance, as well as higher safety and reliability standards [1~3]. Furthermore, with the rapid development and wide application of communication networks, the FDI problem for networked control systems (NCSs) has also received much attention recently [4~7]. It is noted that models of NCSs in the above references were discrete-time system and, therefore, the discretization were needed for NCSs with continuous-time dynamic processes, such that the inter-sampling behaviors were not taken into account. When the network-induced delay is unknown but bounded, the continuous-time system model proposed in references [8, 9] are more reasonable. To authors' best knowledge, however, Zhong and Han [10] studied the problem of fault detection filter design in the continuous-time domain, which is on the assumption of network induced delay (controller-to-actuator) being known. Due to time-delays frequently encountered in practical control systems, and packet dropouts and packets out of sequence is also unavoidable, the problem of fault detection for NCSs is still open and remains challenging, which is the main motivation of this study.

In this paper, we will design the fault detection filter for a class of networked control systems. Under the assumptions of network-induced time delay being unknown but bounded, packet dropouts and packets out of sequence being unavoidable, the FDF design will be investigated in the continuous-time domain.

2. PROBLEM FORMULATION

Consider an NCS with plant, actuators, sensors, a controller and an FDF, which is depicted in Figure 1. The plant is a continuous-time linear time-invariant (LTI) process, which can be expressed by

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$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_d d(t) + B_f f(t), \\ y(t) = Cx(t), \\ x(t_0) = x_0, t_0 \ge 0. \end{cases} \tag{1}$$

Where $x(t) \in R^n$, $u(t) \in R^{n_u}$, $y(t) \in R^{n_y}$, $d(t) \in R^{n_d}$, $f(t) \in R^{n_f}$ are the state vector, control input vector, measured output vector, L_2 -norm bounded unknown input and the fault to be detected respectively. $x_0 \in R^n$ denotes the initial condition; A、B、C、B_d and B_f are known matrices with appropriate dimensions. τ_k^{sc} ,τ_k^c and τ_k^{ca} is the sensor-to-observer delay, the controller computational delay and the controller-to-actuator delay respectively. The time delay from instant $i_k h$ when sensor nodes sample sensor data from plant to the instant when the actuator transfer data to the plant is $\tau_k = \tau_k^{ca} + \tau_k^c + \tau_k^{sc}$, where h is the sampling period, $i_k (k = 1, 2, \cdots)$ is an integer, $\{i_1, i_2, i_3, \ldots\} \subset \{0, 1, 2, \ldots\}$ and $\bigcup_{k=1}^{\infty} [i_k h + \tau_k , i_{k+1} h + \tau_{k+1}) = [t_0, \infty)$.

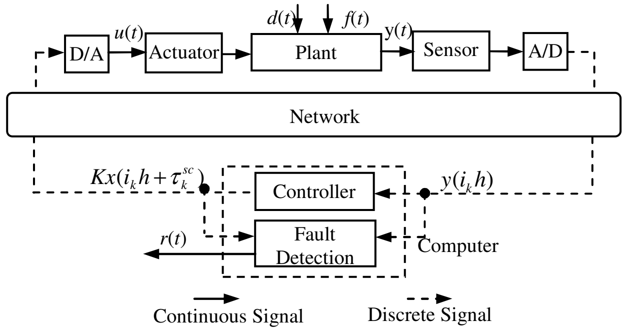

Fig. 1. The structure of an NCS.

It is assumed that the actuator and controller are event driven, the sensor is time driven, and the data are transmitted with a single packet. The real input $u(t)$ and the Fault detection unit input $v(t)$ is realized through a zero-order hold in (1) and given by

$$\begin{cases} u(t) = Kx(i_k h), \quad t \in [i_k h + \tau_k , i_{k+1} h + \tau_{k+1}), \\ \psi(t) = y(i_k h), \quad v(t) = Kx(i_k h), \quad t \in [i_k h + \tau_k^{sc}, i_{k+1} h + \tau_{k+1}^{sc}), \end{cases} \tag{2}$$

K is the known state feedback matrix. As pointed out in references [8, 9], $i_{k+1} > i_k$ is not required, and $i_{k+1} > i_k$ means that the data sequence received by the actuator is the same as that of the sensor sampling data. $\{i_1, i_2, i_3, \ldots\} = \{0, 1, 2, \ldots\}$ means that no packet dropout occurs in the data transmission. If $i_{k+1} = i_k + 1$, it implies that $\tau_{k+1} + h > \tau_k$, which includes $\tau_k < h$ and $\tau_k = \hat{\tau}$ as special cases, where $\hat{\tau}$ is a constant.

Throughout this paper, it is also assumed that there exist constants $\tau_{m1} \ge 0,\quad \tau_{m2} \ge 0$ and $\eta \ge 0$ such that

$$\begin{cases} \tau_k^{sc} \ge \tau_{m1}, \tau_k^{ca} \ge \tau_{m2}, (i_{k+1} - i_k)h + \tau_{k+1}^{sc} \le \eta, \\ i_k h + \tau_k \le i_{k+1} h + \tau_{k+1}^{sc}, \end{cases} \tag{3}$$

Then we have

$$\tau_k \ge \tau_{m1} + \tau_{m2}, \quad \tau_k \le \eta, \quad \tau_k^{sc} \le \eta - \tau_{m2}, \quad \tau_k^{ca} \le \eta - \tau_{m1}, \quad (i_{k+1} - i_{k-1})h + \tau_{k+1}^{sc} \le 2\eta - \tau_{m1}, \tag{4}$$

In order to detect the fault, we propose to design an observer-based FDF for the NCS through an H_∞ filtering formulation in the continuous-time domain such that

$$\sup_{w \in L_2, \|w\|_2 \ne 0} \frac{\left\| r - W_f(s) f \right\|_2}{\|w\|_2} \le \gamma \tag{5}$$

where $\gamma > 0$ is a prescribed scalar, $r(t)$ is the generated residual, $w(t) = \begin{bmatrix} d^{\mathrm{T}}(t) & f^{\mathrm{T}}(t) \end{bmatrix}^{\mathrm{T}}$, $W_f(s) \in RH_\infty$ is a weighting matrix used to limit the frequency interval of the fault being interested. Let

$$\begin{cases} \dot{x}_f(t) = A_{wf} x_f(t) + B_{wf} f(t), \\ r_f(t) = C_{wf} x_f(t), \\ x_f(t_0) = x_{f0}, \end{cases}$$

be a minimal state space realization of $W_f(s)$, $x_f(t) \in R^{n_w}$ is the state vector, x_{f0} denotes the initial condition, A_{wf}, B_{wf}, C_{wf} are known matrices with appropriate dimensions. In this paper, an observer-based FDF is proposed as the residual generator, which is described by

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bv(t) + H_1(\psi(t) - \hat{\psi}(t)), \\ \dot{\hat{x}}_f(t) = A_{wf}\hat{x}_f(t) + H_2(\psi(t) - \hat{\psi}(t)), \\ \hat{\psi}(t) = C\hat{x}(t), \\ r(t) = C_{wf}\hat{x}_f(t), \end{cases} \tag{6}$$

with $\hat{x}(t) \in R^n$ and $\hat{x}_f(t) \in R^{n_w}$ are the estimation of the state $x(t)$ and $x_f(t)$ respectively, $r(t)$ is the residual signal, H_1 and H_1 are the FDF gain matrix to be determined. It follows from (2)-(6) that

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + BKx(i_k h) + H_1(y(i_k h) - C\hat{x}(t)), \\ \dot{\hat{x}}_f(t) = A_{wf}\hat{x}_f(t) + H_2(y(i_k h) - C\hat{x}(t)), \\ r(t) = C_{wf}\hat{x}_f(t), t \in [i_k h + \tau_k^{sc}, \quad i_{k+1}h + \tau_{k+1}^{sc}), \quad k = 1, 2, \ldots \end{cases}$$

Defining $e(t) = x(t) - \hat{x}(t)$, $e_f(t) = x_f(t) - \hat{x}_f(t)$, $r_e(t) = r_f(t) - r(t)$, we have

$$\begin{cases} \dot{\xi}(t) = (A_\xi - H_\xi C_\xi)\xi(t) + B_{\xi 1}K_\xi \xi(i_{k-1}h) - B_{\xi 2}K_\xi \xi(i_k h) - H_\xi C_{\xi 1}\xi(i_k h) + B_{\xi w}w(t), \\ r_e(t) = C_{\xi f}\xi(t), \quad t \in [i_k h + \tau_k^{sc}, \quad i_k h + \tau_k), \end{cases} \tag{7}$$

$$\begin{cases} \dot{\xi}(t) = (A_\xi - H_\xi C_\xi)\xi(t) + B_\xi K_\xi \xi(i_k h) - H_\xi C_{\xi 1}\xi(i_k h) + B_{\xi w}w(t), \\ r_e(t) = C_{\xi f}\xi(t), \quad t \in [i_k h + \tau_k, \quad i_{k+1}h + \tau_{k+1}^{sc}), \end{cases} \tag{8}$$

where

$$\xi(t) = \begin{bmatrix} x(t) \\ e(t) \\ e_f(t) \end{bmatrix}, A_\xi = \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A_{wf} \end{bmatrix}, H_\xi = \begin{bmatrix} 0 \\ H_1 \\ H_2 \end{bmatrix}, B_{\xi 1} = \begin{bmatrix} B \\ B \\ 0 \end{bmatrix}, B_{\xi 2} = \begin{bmatrix} 0 \\ B \\ 0 \end{bmatrix}, B_\xi = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, B_{\xi w} = \begin{bmatrix} B_d & B_f \\ B_d & B_f \\ 0 & B_{wf} \end{bmatrix}$$

$$C_\xi = \begin{bmatrix} -C & C & 0 \end{bmatrix}, \ C_{\xi 1} = \begin{bmatrix} C & 0 & 0 \end{bmatrix}, C_{\xi f} = \begin{bmatrix} 0 & 0 & C_{wf} \end{bmatrix}, K_\xi = \begin{bmatrix} K & 0 & 0 \end{bmatrix},$$

The design of H_∞-FDF for NCS can be re-formulated as to find H_1 and H_1 such that system (7)-(8) with $w(t) \equiv 0$ is asymptotically stable and satisfies

$$\sup_{w \in L_2, \|w\|_2 \neq 0} \frac{\|r_e\|_2}{\|w\|_2} \leq \gamma. \tag{9}$$

3. DESIGN OF THE H_∞-FDF FOR NCS

In this section, by applying the Lyapunov-Krasovskii functional approach, we will concentrate our attention on the delay-dependent sufficient condition to solve the H_∞-FDF problem, and obtain a solution of H_1 and H_2. Following lemma is required in the later development.

Lemma 1[11]: For any constant matrix $M \in R^{n\times n}$, $M = M^T > 0$, scalar $\tau > 0$, and vector function $\dot{x} : [-\tau, 0] \to R^n$ such that the following integration is well defined

$$-\tau\int_{t-\tau}^{t} \dot{x}^T(s)M\dot{x}(s)ds \le \begin{bmatrix} x^T(t) & x^T(t-\tau) \end{bmatrix}\begin{bmatrix} -M & M \\ M & -M \end{bmatrix}\begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix}.$$

From Lemma 1, the delay-dependent sufficient condition to the H_∞-FDF problem is given in thereom 1, and the solution of H_1 and H_2 is given in the algorithm 1.

Theorem 1: For given scalar $\gamma > 0$, $\eta \ge 0$ and $\tau_{mi} \ge 0\ (i = 1,2)$, the H_∞-FDF problem depicted by (9) is solvable, if there exist real matrices P, Q_i, R_i, S_i $(i = 1,2)$ and matrices H_ξ such that

$$\tilde{\Psi} = \begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & PB_{\xi w} & \tau_{av1}\left(A_\xi - H_\xi C_\xi\right)^{\mathrm{T}} \\ * & -Q_1 - R_1 & 0 & 0 & 0 & 0 & \tau_{av1}\left(B_{\xi 1}K_\xi\right)^{\mathrm{T}} \\ * & * & -Q_2 - R_2 & 0 & 0 & 0 & \tau_{av1}\left(-B_{\xi 2}K_\xi - H_\xi C_{\xi 1}\right)^{\mathrm{T}} \\ * & * & * & -S_1 & 0 & 0 & \tau_{av1}\left(B_{\xi 1}K_\xi\right)^{\mathrm{T}} \\ * & * & * & * & -S_2 & 0 & \tau_{av1}\left(-B_{\xi 2}K_\xi - H_\xi C_{\xi 1}\right)^{\mathrm{T}} \\ * & * & * & * & * & -\gamma^2 I & \tau_{av1}B_{\xi w}^{\mathrm{T}} \\ * & * & * & * & * & * & -R_1^{-1} \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \\ * & * & * & * & * & * & * \end{bmatrix} \rightarrow$$

$$\leftarrow \begin{bmatrix} \tau_{av2}\left(A_\xi - H_\xi C_\xi\right)^{\mathrm{T}} & \sqrt{2}\delta_1\left(A_\xi - H_\xi C_\xi\right)^{\mathrm{T}} & \sqrt{2}\delta_2\left(A_\xi - H_\xi C_\xi\right)^{\mathrm{T}} \\ \tau_{av2}\left(B_{\xi 1}K_\xi\right)^{\mathrm{T}} & \sqrt{2}\delta_1\left(B_{\xi 1}K_\xi\right)^{\mathrm{T}} & \sqrt{2}\delta_2\left(B_{\xi 1}K_\xi\right)^{\mathrm{T}} \\ \tau_{av2}\left(-B_{\xi 2}K_\xi - H_\xi C_{\xi 1}\right)^{\mathrm{T}} & \sqrt{2}\delta_1\left(-B_{\xi 2}K_\xi - H_\xi C_{\xi 1}\right)^{\mathrm{T}} & \sqrt{2}\delta_2\left(-B_{\xi 2}K_\xi - H_\xi C_{\xi 1}\right)^{\mathrm{T}} \\ \tau_{av2}\left(B_{\xi 1}K_\xi\right)^{\mathrm{T}} & \sqrt{2}\delta_1\left(B_{\xi 1}K_\xi\right)^{\mathrm{T}} & \sqrt{2}\delta_2\left(B_{\xi 1}K_\xi\right)^{\mathrm{T}} \\ \tau_{av2}\left(-B_{\xi 2}K_\xi - H_\xi C_{\xi 1}\right)^{\mathrm{T}} & \sqrt{2}\delta_1\left(-B_{\xi 2}K_\xi - H_\xi C_{\xi 1}\right)^{\mathrm{T}} & \sqrt{2}\delta_2\left(-B_{\xi 2}K_\xi - H_\xi C_{\xi 1}\right)^{\mathrm{T}} \\ \tau_{av2}B_{\xi w}^{\mathrm{T}} & \sqrt{2}\delta_1 B_{\xi w}^{\mathrm{T}} & \sqrt{2}\delta_2 B_{\xi w}^{\mathrm{T}} \\ 0 & 0 & 0 \\ -R_2^{-1} & 0 & 0 \\ * & -S_1^{-1} & 0 \\ * & * & -S_2^{-1} \end{bmatrix} < 0$$

$$(1,1) = \left(A_\xi - H_\xi C_\xi\right)^{\mathrm{T}} P + P\left(A_\xi - H_\xi C_\xi\right) + \sum_{i=1}^{2}\left(Q_i - R_i\right) + C_\xi^{\mathrm{T}} C_\xi,\ (1,2) = PB_{\xi 1}K_\xi + R_1,\ (1,3) = P\left(-B_{\xi 2}K_\xi - H_\xi C_{\xi 1}\right) + R_2,$$

$$(1,4) = PB_{\xi 1}K_\xi,\ (1,5) = P\left(-B_{\xi 2}K_\xi - H_\xi C_{\xi 1}\right),\ \tau_{av1} = \frac{1}{2}\left(\tau_{m2} + 2\eta\right),\ \sigma_1 = \frac{1}{2}\left(2\eta - 2\tau_{m1} - \tau_{m2}\right),\ \tau_{av2} = \frac{1}{2}\left(\tau_{m1} + \eta\right),\ \sigma_2 = \frac{1}{2}\left(\eta - \tau_{m1}\right).$$

Proof: Choose a Lyapunov-Krasovskii functional candidate as

$$\begin{aligned} V(t) = {} & \xi^{\mathrm{T}}(t)P\xi(t) + \sum_{i=1}^{2}\int_{t-\tau_{avi}}^{t} \xi^{\mathrm{T}}(\theta)Q_i\xi(\theta)d\theta + \sum_{i=1}^{2}\int_{-\tau_{avi}}^{0} ds\int_{t+s}^{t} \dot{\xi}^{\mathrm{T}}(\theta)(\tau_{avi}R_i)\dot{\xi}(\theta)d\theta \\ & + \sum_{i=1}^{2}\int_{-\tau_{avi}-\sigma_i}^{-\tau_{avi}+\sigma_i} ds\int_{t+s}^{t} \dot{\xi}^{\mathrm{T}}(\theta)(\sigma_i S_i)\dot{\xi}(\theta)d\theta, \end{aligned} \tag{10}$$

where $P > 0$, $Q_i > 0$, $R_i > 0$, $S_i > 0$ $(i = 1,2)$. From lemma 1, we can get the main result of this paper. Due to limited space, the detailed process is omitted here.

Then we are in the position to present the solution of the FDF gain matrices H_1 and H_2. Let

$$P=\begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix},\quad PH_\xi=\begin{bmatrix} 0 \\ Y_\xi \end{bmatrix}=Y,$$

where $P_1\in R^{n\times n}$, $P_2\in R^{(n+n_w)\times(n+n_w)}$, $Y_\xi\in R^{(n+n_w)\times n_y}$. Pre- and post-multiplying both side of $\widetilde{\Psi}<0$ with $diag(I,I,I,I,I,I,P,P,P,P)$, we can get matrix inequality $\Gamma<0$.

$$\Gamma=\begin{bmatrix}
\Gamma_{(1,1)} & PB_{\xi1}K_\xi+R_1 & -PB_{\xi2}K_\xi-YC_{\xi1}+R_2 & PB_{\xi1}K_\xi & -PB_{\xi2}K_\xi-YC_{\xi1} & PB_{\xi w} & \tau_{av1}\left(PA_\xi-YC_\xi\right)^{\mathrm{T}} & \tau_{av2}\left(PA_\xi-YC_\xi\right)^{\mathrm{T}} & \sqrt{2}\delta_1\left(PA_\xi-YC_\xi\right)^{\mathrm{T}} & \sqrt{2}\delta_2\left(PA_\xi-YC_\xi\right)^{\mathrm{T}} \\
* & -Q_1-R_1 & 0 & 0 & 0 & 0 & \tau_{av1}\left(B_{\xi1}K_\xi\right)^{\mathrm{T}} & \tau_{av2}\left(B_{\xi1}K_\xi\right)^{\mathrm{T}} & \sqrt{2}\delta_1\left(B_{\xi1}K_\xi\right)^{\mathrm{T}} & \sqrt{2}\delta_2\left(B_{\xi1}K_\xi\right)^{\mathrm{T}} \\
* & * & -Q_2-R_2 & 0 & 0 & 0 & \tau_{av1}\left(-PB_{\xi2}K_\xi-YC_{\xi1}\right)^{\mathrm{T}} & \tau_{av2}\left(-PB_{\xi2}K_\xi-YC_{\xi1}\right)^{\mathrm{T}} & \sqrt{2}\delta_1\left(-PB_{\xi2}K_\xi-YC_{\xi1}\right)^{\mathrm{T}} & \sqrt{2}\delta_2\left(-PB_{\xi2}K_\xi-YC_{\xi1}\right)^{\mathrm{T}} \\
* & * & * & -S_1 & 0 & 0 & \tau_{av1}\left(PB_{\xi1}K_\xi\right)^{\mathrm{T}} & \tau_{av1}\left(PB_{\xi1}K_\xi\right)^{\mathrm{T}} & \sqrt{2}\delta_1\left(PB_{\xi1}K_\xi\right)^{\mathrm{T}} & \sqrt{2}\delta_2\left(PB_{\xi1}K_\xi\right)^{\mathrm{T}} \\
* & * & * & * & -S_2 & 0 & \tau_{av1}\left(-PB_{\xi2}K_\xi-YC_{\xi1}\right)^{\mathrm{T}} & \tau_{av2}\left(-PB_{\xi2}K_\xi-YC_{\xi1}\right)^{\mathrm{T}} & \sqrt{2}\delta_1\left(-PB_{\xi2}K_\xi-YC_{\xi1}\right)^{\mathrm{T}} & \sqrt{2}\delta_2\left(-PB_{\xi2}K_\xi-YC_{\xi1}\right)^{\mathrm{T}} \\
* & * & * & * & * & -\gamma^2 I & \tau_{av1}B_{\xi w}^{\mathrm{T}}P & \tau_{av2}B_{\xi w}^{\mathrm{T}}P & \sqrt{2}\delta_1 B_{\xi w}^{\mathrm{T}}P & \sqrt{2}\delta_2 B_{\xi w}^{\mathrm{T}}P \\
* & * & * & * & * & * & -PR_1^{-1}P & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -PR_2^{-1}P & 0 & 0 \\
* & * & * & * & * & * & * & * & -PS_1^{-1}P & 0 \\
* & * & * & * & * & * & * & * & * & -PS_2^{-1}P
\end{bmatrix}$$

$$\Gamma_{(1,1)}=A_\xi^{\mathrm{T}}P-C_\xi^{\mathrm{T}}Y^{\mathrm{T}}+PA_\xi-YC_\xi+\sum_{i=1}^{2}\left(Q_i-R_i\right)+C_\xi^{\mathrm{T}}C_\xi$$

Notice that the inequality $\Gamma<0$ includes nonlinear terms $PR_i^{-1}P$ and $PS_i^{-1}P$ $(i=1,2)$, introducing new variables $R_{pi}>0,S_{pi}>0$ $(i=1,2)$ such that

$$R_{p1}\le PR_1^{-1}P,\ R_{p2}\le PR_2^{-1}P,\ S_{p1}\le PS_1^{-1}P,\ S_{p2}\le PS_2^{-1}P. \tag{11}$$

we can see that $\Gamma<0$ is feasible if $\widetilde{\Gamma}<0$ (13) is satisfied. According to Schur complement, matrix inequalities in (11) are equivalent to

$$\begin{bmatrix} R_{pi}^{-1} & P^{-1} \\ P^{-1} & R_i^{-1} \end{bmatrix}\ge 0,\quad \begin{bmatrix} S_{pi}^{-1} & P^{-1} \\ P^{-1} & S_i^{-1} \end{bmatrix}\ge 0,\quad i=1,2\ .$$

Introduce again the following new variables $\widetilde{P}=P^{-1},\widetilde{S}_i=S_i^{-1},\widetilde{R}_i=R_i^{-1},\widetilde{R}_{Pi}=R_{pi}^{-1},\widetilde{S}_{Pi}=S_{pi}^{-1},\ (i=1,2)$, Using the idea in a cone complementary linearization algorithm in [12], the problem of finding a feasible solution to matrix inequality $\Gamma<0$ can be formulated as the following minimization problem

$$\min_{P,Q_i,R_i,S_i,Y,R_{pi},S_{pi},\widetilde{P},\widetilde{R}_i,\widetilde{S}_i,\widetilde{R}_{pi},\widetilde{S}_{pi}}\ \mathrm{Trace}\left(P\widetilde{P}+\sum_{i=1}^{2}(R_i\widetilde{R}_i+S_i\widetilde{S}_i+R_{Pi}\widetilde{R}_{Pi}+S_{Pi}\widetilde{S}_{Pi})\right),\ \text{subject to LMIs (12).}$$

$$\begin{cases} \tilde{\Gamma} < 0, \quad \begin{bmatrix} P & I \\ I & \tilde{P} \end{bmatrix} \ge 0, \quad \begin{bmatrix} R_i & I \\ I & \tilde{R}_i \end{bmatrix} \ge 0, \quad \begin{bmatrix} S_i & I \\ I & \tilde{S}_i \end{bmatrix} \ge 0, \\ \begin{bmatrix} R_{Pi} & I \\ I & \tilde{R}_{Pi} \end{bmatrix} \ge 0, \quad \begin{bmatrix} S_{Pi} & I \\ I & \tilde{S}_{Pi} \end{bmatrix} \ge 0, \quad \begin{bmatrix} \tilde{R}_{Pi} & \tilde{P} \\ \tilde{P} & \tilde{R}_i \end{bmatrix} \ge 0, \\ \begin{bmatrix} \tilde{S}_{Pi} & \tilde{P} \\ \tilde{P} & \tilde{S}_i \end{bmatrix} \ge 0, \quad i = 1,2. \end{cases} \tag{12}$$

which can be solved by the following algorithm, moreover the FDF gain matrices H_1 and H_2 can be given by $H_\xi = P^{-1}Y = \begin{bmatrix} 0 & H_1^{\mathrm{T}} & H_2^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$.

$$\tilde{\Gamma} = \left[\begin{array}{cccccccccc}
\Gamma_{(1,1)} & PB_{\xi1}K_\xi + R_1 & -PB_{\xi2}K_\xi - YC_{\xi1} + R_2 & PB_{\xi1}K_\xi & -PB_{\xi2}K_\xi - YC_{\xi1} & PB_{\xi w} + C_{\xi f}^{\mathrm{T}} D_{\xi f} & \tau_{av1}\left(PA_\xi - YC_\xi\right)^{\mathrm{T}} & \tau_{av2}\left(PA_\xi - YC_\xi\right)^{\mathrm{T}} & \sqrt{2}\delta_1\left(PA_\xi - YC_\xi\right)^{\mathrm{T}} & \sqrt{2}\delta_2\left(PA_\xi - YC_\xi\right)^{\mathrm{T}} \\
* & -Q_1 - R_1 & 0 & 0 & 0 & 0 & \tau_{av1}\left(B_{\xi1}K_\xi\right)^{\mathrm{T}} & \tau_{av2}\left(B_{\xi1}K_\xi\right)^{\mathrm{T}} & \sqrt{2}\delta_1\left(B_{\xi1}K_\xi\right)^{\mathrm{T}} & \sqrt{2}\delta_2\left(B_{\xi1}K_\xi\right)^{\mathrm{T}} \\
* & * & -Q_2 - R_2 & 0 & 0 & 0 & \tau_{av1}\left(-PB_{\xi2}K_\xi - YC_{\xi1}\right)^{\mathrm{T}} & \tau_{av2}\left(-PB_{\xi2}K_\xi - YC_{\xi1}\right)^{\mathrm{T}} & \sqrt{2}\delta_1\left(-PB_{\xi2}K_\xi - YC_{\xi1}\right)^{\mathrm{T}} & \sqrt{2}\delta_2\left(-PB_{\xi2}K_\xi - YC_{\xi1}\right)^{\mathrm{T}} \\
* & * & * & -S_1 & 0 & 0 & \tau_{av1}\left(PB_{\xi1}K_\xi\right)^{\mathrm{T}} & \tau_{av1}\left(PB_{\xi1}K_\xi\right)^{\mathrm{T}} & \sqrt{2}\delta_1\left(PB_{\xi1}K_\xi\right)^{\mathrm{T}} & \sqrt{2}\delta_2\left(PB_{\xi1}K_\xi\right)^{\mathrm{T}} \\
* & * & * & * & -S_2 & 0 & \tau_{av1}\left(-PB_{\xi2}K_\xi - YC_{\xi1}\right)^{\mathrm{T}} & \tau_{av2}\left(-PB_{\xi2}K_\xi - YC_{\xi1}\right)^{\mathrm{T}} & \sqrt{2}\delta_1\left(-PB_{\xi2}K_\xi - YC_{\xi1}\right)^{\mathrm{T}} & \sqrt{2}\delta_2\left(-PB_{\xi2}K_\xi - YC_{\xi1}\right)^{\mathrm{T}} \\
* & * & * & * & * & -\gamma^2 I + D_{\xi f}^{\mathrm{T}} D_{\xi f} & \tau_{av1}B_{\xi w}^{\mathrm{T}}P & \tau_{av2}B_{\xi w}^{\mathrm{T}}P & \sqrt{2}\delta_1 B_{\xi w}^{\mathrm{T}}P & \sqrt{2}\delta_2 B_{\xi w}^{\mathrm{T}}P \\
* & * & * & * & * & * & -R_{P1} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -R_{P2} & 0 & 0 \\
* & * & * & * & * & * & * & * & -S_{P1} & 0 \\
* & * & * & * & * & * & * & * & * & -S_{P2}
\end{array}\right] \tag{13}$$

From the above analysis, we can get the following algorithm to the solution of the FDF gain matrices.

Algorithm 1:

Step 1. For giving scalars $\gamma > 0$, $\eta \ge 0$, $\tau_{mi} \ge 0 (i = 1,2)$ and the iteration number $K_{\max} > 0$, find a feasible solution to LMIs (12) which is denoted as

$(P_0, Q_{i0}, R_{i0}, S_{i0}, Y_0, R_{pi0}, S_{pi0}, \tilde{P}_0, \tilde{R}_{i0}, \tilde{S}_{i0}, \tilde{R}_{pi0}, \tilde{S}_{pi0})$, ($i = 1,2$). If there is none, exit. Otherwise, set $k = 0$.

Step 2. Solve the following minimization problem

$$\text{Minimize } \mathrm{Trace}\left(P\tilde{P}_k + \tilde{P}P_k + \sum_{i=1}^{2} \begin{pmatrix} R_i\tilde{R}_{ik} + \tilde{R}_i R_{ik} + S_i\tilde{S}_{ik} + \tilde{S}_i S_{ik} \\ + R_{Pi}\tilde{R}_{Pik} + S_{Pi}\tilde{S}_{Pik} + \tilde{S}_{Pi}S_{Pik} \end{pmatrix} \right) \text{subject to LMIs (12).}$$

Obtain $P, Q_i, R_i, S_i, Y, R_{pi}, S_{pi}, \tilde{P}, \tilde{R}_i, \tilde{S}_i, \tilde{R}_{pi}, \tilde{S}_{pi}$ $(i = 1,2)$, and set

$$\begin{aligned} & P_{k+1} = P,\ R_{i(k+1)} = R_i,\ S_{i(k+1)} = S_i,\ R_{pi(k+1)} = R_{pi}, \\ & S_{pi(k+1)} = S_{pi},\ \tilde{R}_{i(k+1)} = \tilde{R}_i,\ \tilde{S}_{i(k+1)} = \tilde{S}_i, \\ & \tilde{R}_{pi(k+1)} = \tilde{R}_{pi},\ \tilde{S}_{pi(k+1)} = \tilde{S}_{pi}, \quad i = 1,2. \end{aligned}$$

Step 3. Let $k = k+1$. Repeat the operation in step 2 till the matrix inequality $\Gamma < 0$ is satisfied or $k > K_{\max}$.

4. A Numerical Example

The continuous-time LTI process is described by

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(t) + \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} d(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} f(t), \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t). \end{cases}$$

Set $K = \begin{bmatrix} -3.75 & -11.5 \end{bmatrix}$, $h = 1$ sec, $\eta = 1.25$ sec, $\tau_{m1} = 0.02$ sec, $\tau_{m2} = 0.01$ sec, a minimal state space realization of $W_f(s) = \dfrac{1}{s^2 + s + 0.27}$ is

$$\begin{cases} \dot{x}_f(t) = \begin{bmatrix} 0 & 1 \\ -0.27 & -1 \end{bmatrix} x_f(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f(t), \\ r_f(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_f(t). \end{cases}$$

Let $\gamma = 0.5$, $K_{\max} = 200$. Applying Algorithm 1, we have

$$H_1 = \begin{bmatrix} 41.641 \\ 10.851 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 30.977 \\ 9.196 \end{bmatrix}.$$

for $t \in [0, 200]$ sec, it is assumed that the unknown input *d*(t) is a band-limited white noise with power 0.1 (zeroth-order hold with sampling time 0.1 sec) a fault signal *f*(t) is shown in figure 2, the generated residual *r*(t) is shown in Figure 3.

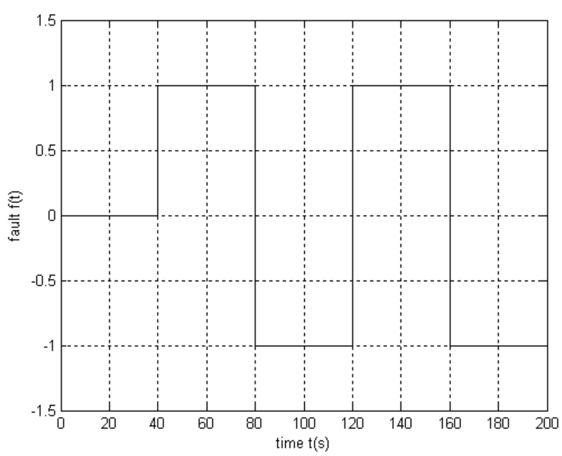

Fig. 2. The fault signal *f*(t).

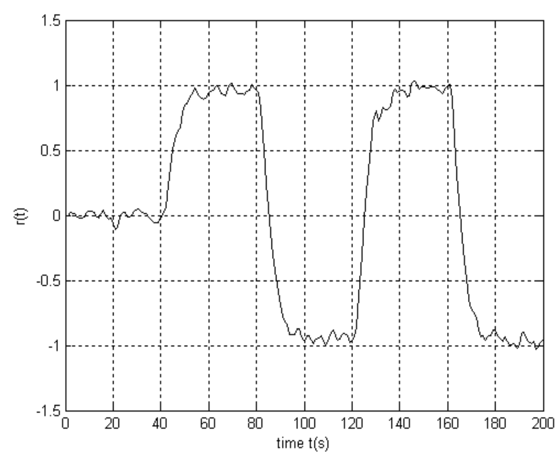

Fig. 3. The residual signal *r*(t).

From the simulation result we can see the effectiveness of the proposed method.

5. Summary

FDF design problem for NCSs has been investigated in this paper. Based on the model of an NCS in the continuous-time domain, we apply the H_∞ estimation of *W(f)f*(*s*) as the residual signal. Furthermore, a delay-dependent sufficient condition on the existence of the problem is derived by using the Lyapunov-Krasovskii functional approach, an algorithm is proposed to get a feasible solution to the FDF gain matrices in terms of LMIs by using a cone complementary technology. A simulation example is given to demonstrate the effectiveness of the proposed method.

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