# Continuous Learning Support Vector Machine to Estimate Stability Lobe Diagrams in Milling

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ABSTRACT

The productivity of milling processes is limited by the occurrence of chatter vibrations. The correlation of the maximum stable cutting depth and the spindle speed can be shown in a stability lobe diagram (SLD). Today it is a great effort to estimate the SLD. Lot's of experiments are necessary to measure the SLD or derive a detailed mathematical model to calculate the SLD. Moreover not only cutting depth, but also the cutting width should be taken into account. This paper presents an approach to learn the multidimensional stability lobe diagram (MSLD) during the production based on continuously measured signals using a support vector machine. The support vector machine is extended to make it capable for continuous learning and time-variant systems. The process conditions are classified as stable or unstable. The learned MSLDs are very similar to the analytically calculated MSLDs. Changes over time in the system dynamics can also be learned by the proposed algorithm.

#### 1. Introduction

Increasing the productivity and performance is one of the most important objectives in todays' machining industries. One of the limiting factors of productivity is chatter. The chatter vibrations can cause bad surface quality or even damage the work piece, the tool, or the machine tool itself. The appearance of chatter vibrations depends on the spindle speed, the machine dynamics, the tool, the material and the depth of cut [1]. To reach maximum productivity the width and depth of cut should be chosen close to the stability limit. Thus it is necessary to provide the information about the stability limit for each tool-material-combination to the machine user.

Regenerative chatter is the main problem limiting the productivity of turning and milling processes. Due to the flexible structure of the machine tool, the interaction between the surface left by the last tooth and the current tooth of the cutting tool leads to self-excited vibrations. Thus this effect can be described as a time-delay system, which time delay is given by the rotation speed of the spindle [2]. Since the late 1950s there have been several studies investigating the effect of regenerative chatter [3] [4]. As the chatter is a feedback with time delay the spindle speed, causing the delay, and the depth of cut, causing the excitation, are the main parameters influencing the stability of the system. All pairs of spindle speed and depth of cut can be classified as stable or unstable. This can be graphically represented in a stability lobe diagram (SLD) where the border between stable and unstable conditions is drawn [5].

There exist several ways to generate the SLDs. Experimentally they can be extracted by doing cuts for each spindle speed with increasing depth of cut. Based on the measured results for each spindle speed the maximum stable depth of cut can be estimated [6]. A similar approach is to cut with constant depth of cut but increasing spindle speed. By analyzing the vibration signal for each depth of cut the stable spindle speeds can be located [7]. Based on a mathematic model of the milling process the SLDs can also be simulated or calculated. For example Zatrain [8] analyzed the results in time and frequency domain. The semi-discretization method [9] is another possibility to analyze the stability of time delayed systems.

Based on the SLD, the spindle speed with the maximum depth of cut can be selected. Budak [10] showed, that the critical depth of cut depends on the width of cut. The SLDs are changing for different widths of cut. To reach maximum material removal rate the SLD has to be extended to find optimal pairs of width and depth of cut [10]. Thus the two dimensional SLDs are only suitable to select the optimal depth of cut for one width of cut.

The main drawback of the methods to estimate the SLD mentioned above is, that there are several measurements necessary to extract the SLD itself or to get the mathematical model that can be used to calculate the SLD analytically.

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Moreover it is known, that the dynamic behavior of machine tools is changing over time [11]. Thus a SLD calculated once is not valid for the whole life of the machine tool and the measurements and calculations have to be repeated periodically.

There exist several techniques and methods to detected chatter during the milling process. Different signals, such as vibration [7], sound [6] or force [12] can be used. The measured time signals can be transformed into frequency domain to detected chatter [1] [13]. Kuljanic et al. [13] proposed a method based on multiple signals using the power spectrum density (PSD). The method used in this paper is similar to that approach but applied on the force normal to the feed direction.

The parameters depth and width of cut are continuously changing due to the geometry of the part and the path of the tool. The change or variation of the spindle speed can be used in unstable process conditions to stabilize the process[14]. The spindle speed can be changed and adapted in real-time according to the process conditions by applying a process controller [15].

In this paper a new approach to learn the SLD during milling using a extended support vector machine (SVM) is presented. As the approach is based on continuous learning there are no experiments necessary. The multidimensional stability lobe diagram (MSLD) is learned during productive milling, taking into account the spindle speed, the cutting depth and the cutting width. The continuous learning guarantees that the MSLD is always up-to-date, even if the dynamic machine behavior is changing. Within an additional parameter optimization loop the MSLD can be used to optimize the parameters for each part in serial or single production.

### 2. LEARNING ALGORITHM AND DATA PROCESSING

### 2.1. SUPPORT VECTOR MACHINE

The support vector machine (SVM) was developed by Cortes and Vapnik [16] as a machine learning method in the early 90s. It is suitable for classification as well as regression analysis. According to the analysis task it is called Support Vector Regression (SVR) or Support Vector Classification (SVC).

The autonomic generation of a stability lobe diagram is a typical classification problem. In the diagram the border between the two different classes stable and unstable are plotted against the milling parameters spindle speed, depth and width of cut.

The basic procedure of the SVC can be shown in binary classification with linearly separable training data. In Figure 1a the training data or instance-label pairs  $(x_i, y_i)$ , i = 1, ..., l are points where  $x_i \in \mathbb{R}^d$ . The two different colours and shaps indicate the class label  $y_i \in \{0,1\}$  of the training points. The algorithm of the SVM calculates a hyper plane of the form given in equation (1), which separates the instance-label pairs under certain boundary conditions.

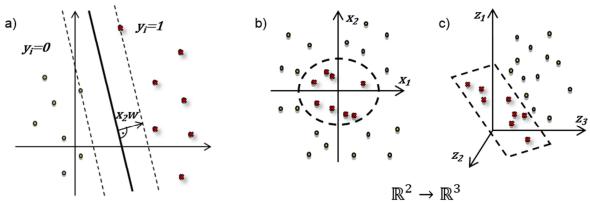


Figure 1. a) Binary classes separated by a hyper plane; b), c) transformation in higher dimensional feature space.

For a set of training data  $\{(x_1, y_1), ..., (x_l, y_l)\} \subset \mathbb{R}^d$  with a domain dimension d of x the parameters w and c of the hyper plane are given by the equation (2).

$$\langle w \cdot x \rangle + c = 0 \tag{1}$$

$$y_i (wx_i + c) - 1 \ge 0, \forall i \in \{1, ..., l\}$$
 (2)

Among all possible hyper planes which separate the classes, the one with the largest margin to the training points of both classes is calculated. Those training points, which are defining this optimal hyper plane, are the so called support vectors. Their distance to the plane can be calculated as  $\frac{1}{\|w^2\|}$ . Adding this boundary condition results in the optimization problem defined by equation (3).

$$\min_{w} \left\{ \frac{1}{2} \|w\|^2 \right\} \qquad \text{subject to} \{ y_i(wx_i + c) \ge 1 \} \forall i \in \{1, ..., l\}. \tag{3}$$

The quadratic optimization problem has a global minimum. Because of noise and outliers it is not possible to separate the training data strictly linear. To overcome this the slip variable  $\xi$  with the weighting C is introduced. Equation (4) describes the extended optimization problem. With an increase of C the tolerance against outliers decreases

$$\min_{w} \left\{ \frac{1}{2} \|w\|^2 + C \sum_{i=1}^{l} \xi_i \right\} \quad \text{subject to} \begin{cases} y_i(wx_i + c) \ge 1, \forall i \in \{1, ..., l\} \\ \xi_i \ge 0, \forall i \in \{1, ..., l\}. \end{cases} \tag{4}$$

The SVM as described previously can only classify linear separable training data. This limitation can be overcome by transforming the training data from the input space to a higher dimensional feature space, where the data is linear separable. An example for that kind of data is shown in Figure 1b. The transformation for this data is given in equation (5).

$$\phi: \mathbb{R}^2 \to \mathbb{R}^3 
(x_1, x_2) \to (z_1, z_2, z_3) := (x_1^2, \sqrt{2}x_1x_2, x_2^2)$$
(5)

The transformed data in  $\mathbb{R}^3$  can be seen in Figure 1c. For more complex problems the transformation  $\phi: X \to F$  can be computed numerically by using different kernel functions [17].

### 2.2. CONTINUOUS LEARNING SVC

As described in section 1 the dynamic behavior of the machine tool can change with time. Thus the learning algorithm has to be capable to deal with time varying systems. The algorithm also has to deal with continuously collected training data because the data is not collected at once by a set of experiments, but all the time during production.

The SVC as it is and as described in section 2.1 is not capable to deal with a continuously increasing set of training data and with time-variant systems. To make the SVC capable for time variant systems and continuous learning it has to be extended to a continuous learning SVC (CSVC).

The SVC method is extended by a data preprocessing algorithm. The entire input domain is discretized into equally sized regions. For each region a fixed limited number of stable and unstable training points are stored. Using the first-in-first-out-principle for each discrete region ensures, that only the latest data is taken into account to learn the current system behavior. In Figure 2 a one-dimensional example is presented. The old data point  $x_{13}$  is deleted, because  $x_{30}$  is a newer one.

To avoid numerical errors during the execution of the SVC the instance-label pairs should be randomized and normalized before starting the learning algorithm. For this normalization the interval [0; 1] was used.

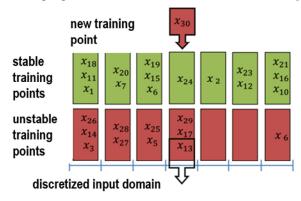


Figure 2. Training data storage for continuous learning strategy.

#### 3. PROCESS MODEL AND ASSESSMENT

## 3,1, MATHEMATIC MODEL OF MILLING DYNAMICS

The chatter effect, on which this paper is focused, can be described as a self-excited vibration. The interaction of the wavy surface of the last cutting edge and the current cutting edge is leading to vibrations. The milling dynamics can be modeled as two spring-and-damper-systems with time delay [18]. Figure 3 shows the model of the dynamics of the up-milling process.

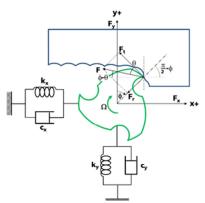


Figure 3. Model of milling dynamics.

The differential equations of the movement of the tool are given in equation (6) and (7).

$$m_x \ddot{x} + c_x \dot{x} + k_x x = F(x(t), x(t-T)) \tag{6}$$

$$m_{\nu}\ddot{y} + c_{\nu}\dot{y} + k_{\nu}y = F(y(t), y(t-T))$$
 (7)

The time delay T is given in equation (8) and depends on the spindle speed S and the number of teeth z of the tool.

$$T = \frac{60}{zS} \tag{8}$$

The process force depends on the radial immersion position  $\phi(t) = 2\pi St$  of the tooth, the axial depth of cut a and the chip thickness  $h(\Phi)$ . With the tangential and radial force coefficients  $(K_t, K_r)$  the force can be divided into a tangential and a radial force  $(F_t, F_r)$  as shown in equation (9) to (12). It is assumed that there is maximum one tooth cutting at a time.

$$F_t = K_t a h(\phi) \tag{9}$$

$$F_r = K_r K_t a h(\phi) \tag{10}$$

$$\theta = tan^{-1} \left( \frac{F_r}{F_r} \right) = tan^{-1} (K_r) \tag{11}$$

$$F(\phi) = K_t \sqrt{1 + K_r^2} ah(\phi) \tag{12}$$

The chip thickness  $h(\Phi)$  can be calculated by using the feed per tooth  $f_Z$ ,  $\Delta x = x(t) - x(t-T)$  and  $\Delta y = y(t) - y(t-T)$  as given in equation (13).

$$h(\phi) = f_z \sin(\phi(t)) + \Delta x \sin(\phi(t)) + \Delta y \cos(\phi(t))$$
(13)

The dynamical model shown in Figure 3 needs the forces  $F_x$  in feed and  $F_y$  in normal direction as the inputs of the spring-and-damper systems. Their calculation is given in equation (14) and (15).

$$F_{r}(\phi) = -F(\phi)\cos(\phi(t) - \theta) \tag{14}$$

$$F_{\nu}(\phi) = F(\phi)\sin(\phi(t) - \theta) \tag{15}$$

## 3.2. PROCESS ASSESSMENT AND CHATTER DETECTION

During cutting, vibrations always appear due to the flexible structure of the machine tool, even under stable process conditions. The frequency of the vibrations varies depending on their origin. In stable cutting conditions the frequency is mainly the tooth pass frequency and their harmonics. The PSD can be used to calculate which frequency has which share in the total vibration energy of the signal. Thus the ratio of the energy E and the chatter Energy  $E_c$  can be used as an indicator for chatter [13].

The PSD of the force signal is calculated based on the Fourier transform of a signal  $S(\omega)$  as given in equation (16). The total energy E in the system is the sum of the periodic energy  $E_p$  coming from the tooth passing, the chatter energy  $E_c$  and the noise energy  $E_n$  (refer to equations (17) and (18)).

$$PSD = |S(\omega)|^2 \tag{16}$$

$$E = \int_0^{+\infty} PSD_S(\omega) \, d\omega \tag{17}$$

$$E = E_p + E_c + E_n \tag{18}$$

The periodic energy  $E_p$  can be calculated as given in equation (19) by calculating the integral in small bands centered in the tooth passing frequency and their harmonics. The energy of noise can be estimated by using the spectrum without cutting (refer to equation (20)).

$$E_p = \sum_{k=1}^{\infty} \left[ \int_{k\omega_t - \delta}^{k\omega_t + \delta} PSD_S(\omega) \, d\omega \right] \tag{19}$$

$$E_n = \int_0^{+\infty} PSD_0(\omega) \, d\omega \tag{20}$$

Thus with equation (18) the chatter indicator  $CI_{ER}$  can be calculated as given in equation (21).

$$CI_{ER} = \frac{E_c}{E} = 1 - \frac{E_p + E_n}{E}$$
 (21)

In Figure 4 the cutting forces of a cut with increasing depth of cut and the calculated  $CI_{ER}$  value is shown. It can be seen, that the  $CI_{ER}$  value can be used as an indicator for the stability of the process.

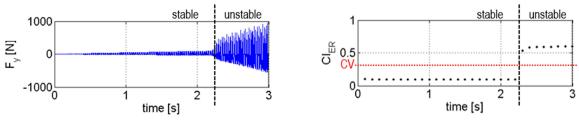


Figure 4. Cutting force and calculated CI<sub>ER</sub>.

By defining a critical value CV for  $CI_{ER}$  as a threshold the process can be classified as stable or unstable. CV was chosen to be 0.37, but as it can be seen in Figure 4 the choice of CV is not critical because the separation between stable and unstable is quite clear.

## 3.3. SIMULATION SETUP

A time domain simulation model is used to show and evaluate the functionalities of the described learning algorithm. The model was parameterized based on the results of experiments. Several cuts in steel with a 20 mm cutter with three inserts were performed and the process forces have been measured. Moreover a modal analysis of the tool was performed to estimate the parameters described in section 3.1. In this application only the tool vibrations are considered, as the work piece is modeled as a rigid block of steel. The signals used for the learning algorithm are most commonly available on machine controls thus the algorithm can also be applied to the real cutting process. In Figure 5a the structure of the model is described.

To learn the MSLD information about the depth and width of cut, the spindle speed and the process stability are necessary. The width and depth of cut can be calculated offline [19] before cutting. The process stability can be analyzed and stored in real-time [12] [13]. In the simulation model all the information about depth and width of cut, the spindle speed and the process stability are available.

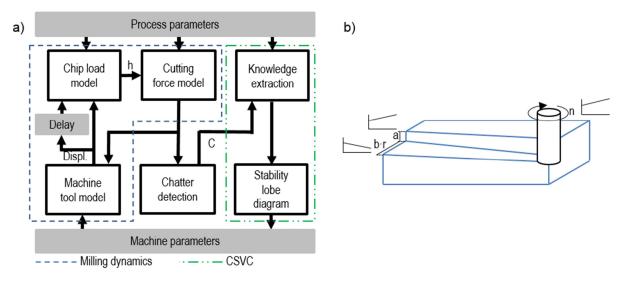


Figure 5. a) Structure of the simulation model; b) single cut.

The learning can be done after the end of the process in non-real-time. The algorithm presented is able to extract all parameters during continuously changing cutting conditions and can assign them to the sets described in section 2. Thus the simulation model is only used to avoid high material and time effort and the learning algorithm as it is can also be applied to a real production process.

Using a simulation model provides the possibility to analytically calculate the SLD for different widths of cut as described in section 1. The results of the analytical solution are compared to the learned MSLD and are used to verify the capability of the learning algorithm.

To simplify the modeling, reduce the memory consumption and simulation effort, single cuts with varying depth of cut were performed. In Figure 5b an example of a possible single cut is presented. The cutting conditions for the simulated tool were chosen as they are suitable for real machining. The spindle speed range was altered from 1300 rev/min to 2500 rev/min. The depth of cut was chosen from a=0 mm to a=5 mm. The width of cut b is defined in relation to the radius b of the tool. That means for b=0 the depth of cut is 0 and for b=1 the depth of cut is equal to the tool radius b. During the simulations b was varied between b=0.2 and b=0.7.

A complete instance-label pair for the CSVM consists of the class value y and a parameter vector x. For the MSLD learning the parameter vector x contains the parameters spindle speed n, depth a and width b of cut. The class value y contains the process stability for these cutting conditions and is 0 for a stable and 1 for an unstable process.

# 4. RESULTS AND VERIFICATION

In this chapter the results of the application of the continuous learning SVC are presented and verified by comparing them to the analytically calculated stability lobe diagram.

To calculate the analytic solution for the SLDs to compare the results with, the 1-DOF approach presented in [20] was chosen. By applying the algorithm for different widths of cuts an analytic MSLD can be calculated. The result of this calculation is presented in Figure 6a. This will be used as reference to compare with the learned MSLD.

The learned MSLD can be seen in Figure 6b. This MSLD was created by applying the CSVC learning algorithm as explained in section 2.2 to the results of the process simulation.

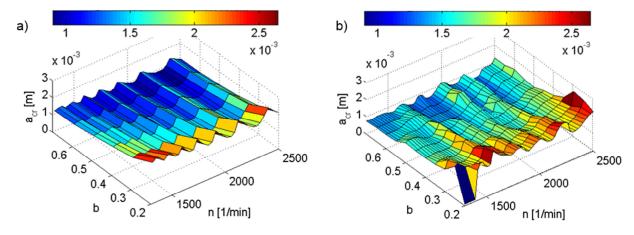


Figure 6. a) Analytically calculated MSLD; b) learned MSLD.

It is obvious that the quality of the learned MSLD improves with an increase of data available for the learning process. Due to the lack of data available at the beginning of the learning process, the learned MSLD has unreliable areas. The development and improvement of the learned MSLD with an increasing number of measured points can be seen in Figure 7.

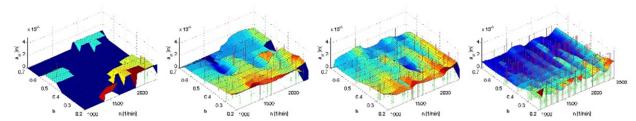


Figure 7. Development of the learned MSLD with increasing amount of data.

The shape and the values for the critical depth of cut  $a_{cr}$  are similar for the learned and the analytic MSLD for the complete range of cutting width b. To make the comparison easier the learned and the calculated SLD for two widths of cut b are shown in Figure 8. The slight shift in the frequencies and amplitude can be explained because of the 2-DOF simulation model for the learning algorithm and the 1-DOF model for the analytic calculation.

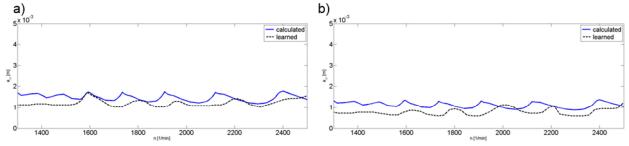


Figure 8. Calculated and learned SLD for b=0.4 (a) and b=0.7 (b).

## 5. CONCLUSION AND OUTLOOK

In this paper a new approach to learn a multidimensional stability lobe diagram (MSLD) based on data gathered during the milling process is presented. The information of the learned MSLD can be used in further process optimization steps to achieve maximum productivity by applying optimal pairs of width and depth of cut.

Using an extended SVM which is able to perform continuous learning enables the collection of training data over time. Thus no additional experiments are necessary to estimate the machine stability, because the data can be collected during productive hours. Moreover the algorithm is able to deal with time-variant system and learns the newest MSLD if the system dynamics are changing.

It has been shown that the algorithm is capable to learn the MSLD taking into account the spindle speed, depth and width of cut. The learned MSLD based on the measured information is similar to the analytically calculated MSLD based on a machine model.

In future research the MSLD can be extended to more parameters such as tool wear or axes position, as the algorithm can also be applied to higher dimensional problems. Moreover the algorithm should be implemented in the control of machine tools, that the SLDs are automatically measured during production. To have a benefit for the productivity an optimization strategy for the program code based on the learned SLDs should be developed.

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