

# User Phase Information Based Inventory Policy for Supply Chain Systems with Remanufacturing

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## ABSTRACT

*Most of the literatures for inventory management policies of supply chain systems with remanufacturing had focused solely on warehouse operation. The existence of its optimal policy has yet been confirmed. In addition, the IT platform nowadays has been well developed so that user phase alignment could be executed at relatively low cost and it is a global phenomenon for companies to pursue cross-tier stretch along its value chain. In this study, a supply chain model comprised of three major biz entities (warehouse, remanufacturing facility and user phase) is presented. An optimal (s,S) type policy for this model is first introduced under the non-leadtime assumption and then extended to the leadtime scenario. The theoretically optimal policy is not applicable as a consequence of lacking information with regard to the failure product's return flow. In order to solve this issue, data mining within the user phase is executed and the specification of user phase information that serves to implementing the optimal policy is articulated. Additionally, numerical experiments are presented to verify the optimality of the (s,S) type policy and the contribution of user phase information sharing to inventory cost reduction.*

## 1. INTRODUCTION

The optimality of (s,S) policy for a broad range of stochastic inventory problems is one of the most essential results in inventory management theories, where s denotes the ordering threshold and S is the order-up-to stock level. Under the assumption that all cost functions are convex, Scarf [1] proved the optimality of (s,S) policy in inventory problems with finite planning horizons. Veinott [2] extended Scarf's work to inventory models with uni-modal holding and penalty costs. Zipkin and Federgruens [3] verified the optimality of (s,S) policy in infinite planning horizon scenarios. Based on the contribution of these scholars, (s,S) policy had become a full-fledged inventory management theory.

With the pervasive penetration of sustainability philosophy in modern industries, immense amount of capital is being invested by various categories of companies to enhance the recyclability within their operation. Increasing number of firms is chasing an integrated business model aiming at salvaging more residual value from the failure products. In order to achieve this, the majority of companies are seeking opportunities of achieving more presence on the user phase through collaborative inventory management tactics or proper recycling incentive schemes. Remanufacturing is becoming an important source to replenish inventory other than procurement. However, no optimal policy has currently been confirmed dealing with this hybrid manufacturing/remanufacturing inventory problems. On the other hand, conventional inventory optimization strategies, including the (s,S) policy, place exclusive emphasis on the warehouse operation, which sometimes are unsuitable nowadays when put into real practice. Hence, the conventional (s,S) type inventory policy, from the inventory management perspective of view, needs to be revised in order to conquer these new business situations.

The research of inventory policies for supply chain systems with remanufacturing dated back to the 1960's and there are basically two primary research lines: deterministic models and stochastic models. Schrady [4] established an EOQ-based remanufacturing inventory model that inspired a series of consequent researches in this area. Nahmias and Rivera [5] published a remanufacturing inventory management model with remanufacturing capacity constraints. Dobos and Richter [6] introduced the disposal option into traditional remanufacturing models that extends the research framework of remanufacturing supply chain systems. Van der Lann and Teunter [7] proposed a heuristic inventory

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management strategy for deterministic supply chain systems with remanufacturing. Along the stochastic research line, Simpson [8] proposed a remanufacturing supply chain management policy within limited planning horizon, and initiated the application of dynamic programming in solving stochastic inventory problems. Inderfurth [9] provided an inventory control strategy that includes the disposal option of returned products under the random demand and recovery assumption and also provided heuristic approaches for acquiring the decision parameters. Kiesmuller [10] brought up a remanufacturing supply chain model based on newsvendor models. Ahiska and King [11] discussed the optimal inventory policy in supply chain systems with a special ordering and remanufacturing pattern.

As we could conclude from existing literatures of inventory policy research for supply chain systems with remanufacturing, only very few papers applied the stochastic periodical review policies. Also, the existence of the optimal inventory policy has yet been confirmed. For the depiction of the user phase demand and the return products, current literatures simply uses certain distributions (e.g. Poission distribution) to fit the inflow and outflow of products from the warehouse managers' standpoint, however, the deficit of referable decision information hinders the implementation of the optimal inventory policy. Hence, here are the issues that would be stressed in this paper: 1) The existence of (s,S) type optimal inventory policy in supply chain systems with remanufacturing. 2) The contribution of user phase information to the implementation of the optimal inventory policy. 3) Specification of the user phase information would be incurred in order to execute the optimal inventory policy.

## 2. INVENTORY POLICY FOR SUPPLY CHAIN SYSTEMS WITH REMANUFACTURING

### 2.1. FRAMEWORK

The stochastic inventory model focuses on the derivation of replenishment decisions over a finite discrete planning horizon under stochastic demands. In this paper, we discuss a supply chain system involving two distinct sources of replenishing the inventory of the serviceable products. Except for traditional procurement, the remanufacturing of failure products is essentially a complementary source of restocking and the quality of remanufacturing products are assumed to be the same as procured ones. Refer to the Figure.1 for the framework of supply chain systems with remanufacturing.

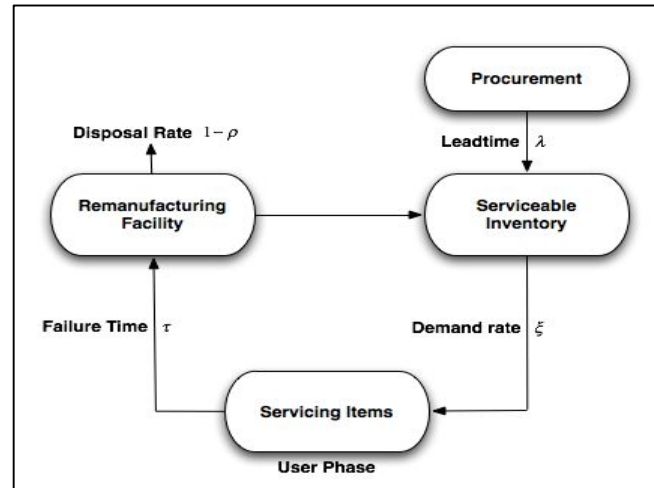


Figure 1. Framework of supply chain systems with remanufacturing.

Following notations are included in the model:

$K$  : Setup cost incurred whenever an order is placed

$c_p$  : Unit purchasing cost

$c_r$  : Unit remanufacturing cost

$h$  : Unit holding cost of serviceable products

$v$  : Unit backorder cost

$\xi_t$  : Stochastic demand in period  $t$

- $r_t$ : Remanufacturing products in period  $t$   
 $p_t$ : Procured products in period  $t$   
 $\lambda$ : Procurement leadtime  
 $\varphi(\cdot)$ : Probability density function  
 $x_t$ : Initial stock level at the beginning of period  $t$   
 $y_t$ : Stock level before demand order is delivered  
 $\mu$ : Remanufacturing leadtime  
 $\rho$ : Remanufacturing rate of used products  
 $\tau$ : Average lifetime of products serving the user phase  
 $N$ : Number of decision periods

The demand over the planning horizon is a sequence of random variables that are not necessarily identically nor independent. We consider an inventory management problem with  $n$  planning periods from  $t=1$  to  $t=n$ . The timeline of pertinent events are as follows:

- 1) At the beginning of the  $t$ th period, the inventory manager faces an initial inventory level of  $x_t$ .
- 2) After checking out the initial inventory, returned products  $r_t$  whose remanufacturing process has been completed would be shipped to the warehouse and the inventory level becomes  $x_t + r_t$ .
- 3) After receiving the remanufacturing products, the inventory manager places an order with quantity  $p_t$  which would be arrived after a fixed procurement leadtime of  $\lambda$  periods. Orders made at the beginning of the  $t$ th period will arrive at the beginning of  $t + \lambda$ th period.
- 4) After issuing the order, outside procurement  $p_{t-\lambda}$  made  $\lambda$  periods ago is delivered and the inventory level becomes  $x_t + r_t + p_{t-\lambda}$ .
- 5) The demand for the current period is realized at the end of the period and then the inventory level becomes  $x_t + r_t + p_{t-\lambda} - \xi_t$ .

We assume that the replenishment of the inventory would be ceased at the end of the planning horizon, so the last order is placed in period  $n - \lambda$ . Without loss of generality, we assume that purchasing cost for inventory is charged at the time of order. The case where purchasing cost is charged at the time of delivery could be transform by a straightforward shift of cost indices. This action has no influence on the inventory level. Excess inventory is carried to the next period, incurring a per-unit holding cost. On the other hand, each unit of unsatisfied demand is backlogged to the next period with a per-unit backlog penalty cost. In the last period, the penalty of lost sales can be accounted through backlog cost.

According to the given timeline of all the pertinent events included in the model, the system dynamics of the inventory is:

$$x_{t+1} = x_t + r_t + p_{t-\lambda} - \xi_t \quad (1)$$

Orders made before period  $t=1$  are pipeline inventories when the planning horizon initiates. Notice that for  $\lambda \geq 1$ , the prior procurement quantities  $p_{t-\lambda}$  ( $t=1, 2, \dots, \lambda$ ) are known values.

For returned products shipped to the remanufacturing facility, only certain percentage could be recycled. Assume arbitrarily that  $1 - \rho$  percentage of returned products have to be disposed due to irreversible failure. In other words, only  $\rho$  percentage of the return products  $re_{t-\mu}$  collected from user phase  $\mu$  periods ago could be remanufactured into serviceable products in period  $t$ . The remanufacturing process for  $re_{t-\mu}$  could be completed and the products would delivered to the warehouse at the beginning of period  $t$  (denoted as  $r_t$ ), such that  $r_t = \rho \cdot re_{t-\mu}$ . Assuming that there is no difference between the procurement and remanufacturing products in serviceable inventory.

A linear cost structure is assumed and all cost parameters are non-negative. A fixed setup cost is incurred whenever an outside procurement is triggered. Purchasing cost in period  $t$  involves a setup cost  $K$  and a unit purchasing cost  $p_t$ :

$$P(p_t) = \begin{cases} K + c_p \cdot p_t & p_t > 0 \\ 0 & p_t = 0 \end{cases} \quad (2)$$

Holding cost of serviceable products in the warehouse is charged proportional to the time and the quantity. Unsatisfied demands are backordered with a linear punish cost which allows the current inventory level to be negative. Holding/backlog cost of serviceable products is charged according to the stock level at the end of the period. This expenditure is proportional to the on-hand stock or unfulfilled demands. Thus, the loss function  $L(\cdot)$  is defined corresponds to two contingencies whether demand exceeded inventory or inventory was sufficient to fulfill the orders in period  $t$ .

$$L(y_t) = h \cdot \int_0^{y_t} (y_t - \xi) \phi(\xi) d\xi + v \cdot \int_{y_t}^{\infty} (\xi - y_t) \phi(\xi) d\xi \quad (3)$$

The loss function is convex since both the holding and penalty costs are convex increasing functions. The remanufacturing cost in period  $t$  is defined as:

$$T(r_t) = c_r \cdot r_t \quad (4)$$

Then, the total cost over  $n$  planning horizon could be formulated as:

$$f = \sum_{t=1}^n [L(y_t) + P(p_t) + T(r_t)] \quad (5)$$

We discretize the planning horizon into  $n$  periods and the inventory control policy established in this paper is on periodic review basis. The objective is to determine the dynamic ordering quantities  $p_t$  so as to minimize the total expected cost over the planning horizon in response to the stochastic demands. Though a deterministic leadtime  $\lambda$  exists between an order is made and delivered, we get started with the non-leadtime case for simplification purposes.

## 2.2. NON-LEADTIME CASE

In this section, dynamic programming is the methodology we implemented here to derive the optimal policy tackling supply chain systems with remanufacturing. In non-leadtime cases, we acquired the full awareness of the quantity of serviceable products  $r_t$  that would be delivered in period  $t$ . The remanufacturing cost  $T(r_t)$  is determined as long as the remanufacturing rate  $\rho$  is fixed. Then the procurement quantity  $p_t$  is the only decision variable we need to take into consideration in the objective function. Hence, the recursive structure of the dynamic programming could be modeled as:

$$f_t(x_t + r_t) = \min_{p_t \geq 0} \left\{ P(p_t) + L(y_t) + \int_0^{\infty} \int_0^{\infty} f_{t+1}(y_t - \xi_t + \tilde{r}_{t+1}) \phi(\xi_t) \phi(\tilde{r}_{t+1}) d\xi_t d\tilde{r}_{t+1} \right\} \quad (6)$$

where  $y_t = x_t + r_t + p_t$ .

To prove the optimality of (s,S) type ordering policy, we first define that:

$$g_t(y_t) = c_p \cdot y_t + L(y_t) + \int_0^{\infty} \int_0^{\infty} f_{t+1}(y_t - \xi_t + \tilde{r}_{t+1}) \phi(\xi_t) \phi(\tilde{r}_{t+1}) d\xi_t d\tilde{r}_{t+1} \quad (7)$$

**Lemma 1** It is economically efficient to make an order from  $x_t$ , if and only if there is some  $y_t > x_t + p_t$  satisfying that  $g_t(x_t + p_t) > K + g_t(y_t)$ . And if such an order is indeed placed, the order-up-to level is the  $y_t$  that minimizes  $g_t(y_t)$ .

The logic behind Lemma 1 is straightforward. Denote where the minimum of  $g_t(\cdot)$  is obtained to be  $S_t$ . If the negative  $g_t(\cdot)$  is uni-modal, finding a unique  $s_t (s_t < S_t)$  satisfying  $g_t(s_t) = g_t(S_t) + K$  indicates that  $(s_t, S_t)$  policy is indeed optimal in period  $t$ . However, further scrutinize of the behavior of  $g_t(\cdot)$  reveals that  $g_t(\cdot)$  actually has a couple of maxima and minima. Thus in this inventory problem of supply chain systems with remanufacturing, we need to prove the optimality of (s,S) type policy by showing that the oscillations along with these maxima and minima are not large enough to cause a deviation from (s,S) type.

**Lemma 2**  $g_t(x) (t = 1, 2, \dots, n)$  are K-convex functions[1].

The necessary and sufficient condition for the optimality of (s,S) type policy is:

$$K + g(a + x) \geq g(x) + ag'(x) (a \geq 0) \quad (8)$$

With the K-convexity of  $g_t(\cdot)$  being confirmed, we obtain  $K + g(a + x) \geq g(x) + ag'(x)$  by letting  $b \rightarrow 0$ . By now, we succeed to demonstrate that (s,S) type ordering policy is indeed optimal in the non-leadtime case.

Denote  $S_t$  as the global minimum of  $g_t(\cdot)$  and  $s_t (s_t < S_t)$  is the point that satisfies  $K + g_t(S_t) = g_t(s_t)$ . We have the following optimal ordering policy in the non-leadtime case.

**Theorem 1**

Ordering policy in non-leadtime case:

If  $x_t + r_t \geq s_t$ , make no order.

If  $x_t + r_t < s_t$ , then order up to  $S_t$ .

In period  $t$ , issue an order to replenish the inventory to  $S_t$  if the stock level after the delivery of remanufacturing product  $x_t + r_t$  is lower than  $s_t$ , make no order otherwise.

**2.3. LEADTIME CASE**

A crucial reason that complicates the ordering process of inventory management problems is the time lag involved in effecting procurement policies. There are situations in which this lag is sufficiently small and it could be disregarded. However, this sort of delay is actually significant in most practical cases and neglecting it would lead to inappropriate inventory strategies. Hence, it is mandate to extend our results to leadtime scenarios. As we have defined, the time lag between placing an order and its subsequent delivery to the warehouse is the procurement leadtime  $\lambda$ .

Notice that the initial stock level  $x_t$  comprises the remanufacturing products of previous periods. Assume that orders  $p_{t-\lambda}$  ( $t = 1, 2, \dots, \lambda$ ) made before period  $t = 1$  are already available for the warehouse manager. The recursive equation of the objective function in the leadtime case could be modeled as follows:

$$f_t(x_t + r_t, p_{t-\lambda}, \dots, p_{t-1}) = \min_{p_t \geq 0} \left\{ P(p_t) + L(x_t + r_t + p_{t-\lambda}) + \int_0^\infty \int_0^\infty f_{t+1}(x_t + r_t + p_{t-\lambda} - \xi_t + \tilde{r}_{t+1}, p_{t-\lambda+1}, \dots, p_{t-1}) \varphi(\xi_t) \varphi(\tilde{r}_{t+1}) d\xi_t d\tilde{r}_{t+1} \right\} \quad (9)$$

**Lemma 3** The objective function could be transformed into following form:

$$\begin{aligned} f_t(x_t + r_t, p_{t-\lambda}, \dots, p_{t-1}) = & \min_{p_t \geq 0} \left\{ P(p_t) + L(x_t + r_t + p_{t-\lambda}) + \int_0^\infty E_{\tilde{r}_{t+1}} [L(x_t + r_t + p_{t-\lambda} - \xi_t + \tilde{r}_{t+1})] \varphi(\xi_t) d\xi_t + \dots \right. \\ & + \int_0^\infty \int_0^\infty E_{\tilde{r}_{t+1}, \tilde{r}_{t+2}} \left[ L\left(x_t + r_t + \sum_{i=0}^1 (p_{t-\lambda+i} - \xi_{t+i} + \tilde{r}_{t+1+i})\right) \right] \varphi(\xi_t) \varphi(\xi_{t+1}) d\xi_t d\xi_{t+1} + \dots \\ & \left. + \int_0^\infty \dots \int_0^\infty E_{\tilde{r}_{t+1}, \dots, \tilde{r}_{t+\lambda}} \left[ L\left(x_t + r_t + \sum_{i=1}^{\lambda-1} (p_{t-\lambda+i} - \xi_{t+i} + \tilde{r}_{t+1+i})\right) \right] \varphi(\xi_t) \dots \varphi(\xi_{t+\lambda}) d\xi_t \dots d\xi_{t+\lambda} + h_t\left(x_t + r_t + \sum_{i=0}^{\lambda-1} p_{t-\lambda+i}\right) \right\} \end{aligned} \quad (10)$$

where  $h_t(z)$  satisfies:

$$h_t(z) = \min_{p_t \geq 0} \left\{ P(p_t) + \int_0^\infty h_{t+1}(z + p_t - \xi_{t+\lambda}) \varphi(\xi_{t+\lambda}) d\xi_{t+\lambda} + \int_0^\infty \dots \int_0^\infty E_{\tilde{r}_{t+1}, \dots, \tilde{r}_{t+\lambda}} \left[ L\left(z + p_t - \sum_{i=0}^{\lambda-1} \xi_{t+i} + \sum_{j=1}^{\lambda} \tilde{r}_{t+j}\right) \right] \varphi(\xi_t) \dots \varphi(\xi_{t+\lambda-1}) d\xi_t \dots d\xi_{t+\lambda-1} \right\} \quad (11)$$

We start our proof of Lemma 3 by taking an inspection into the properties of objective function  $f_t(\cdot)$ , it could be rewrite in following form:

$$\begin{aligned} & f_t(x_t + r_t, p_{t-\lambda}, \dots, p_{t-1}) \\ &= \min_{p_t \geq 0} \left\{ P(p_t) + L(x_t + r_t + p_{t-\lambda}) + \int_0^\infty \int_0^\infty f_{t+1}(x_t + r_t + p_{t-\lambda} - \xi_t + \tilde{r}_{t+1}, p_{t-\lambda+1}, \dots, p_{t-1}) \varphi(\xi_t) \varphi(\tilde{r}_{t+1}) d\xi_t d\tilde{r}_{t+1} \right\} \\ &= L(x_t + r_t + p_{t-\lambda}) + \min_{p_t \geq 0} \left\{ P(p_t) + \int_0^\infty \int_0^\infty f_{t+1}(x_t + p_t + r_t - \xi_t + \tilde{r}_{t+1}, p_{t-\lambda+1}, \dots, p_{t-1}) \varphi(\xi_t) \varphi(\tilde{r}_{t+1}) d\xi_t d\tilde{r}_{t+1} \right\} \end{aligned} \quad (12)$$

Assuming differentiable, its first-order derivative contains a variable set of  $\{x_t + r_t + p_{t-\lambda} + \tilde{r}_{t+1}, p_{t-\lambda+1}, \dots, p_{t-1}\}$ . Hence, when the global minimum is obtained, the optimal ordering quantity  $p_t^*$  is a function that follows the form  $p_t^* = p_t^*(x_t + r_t + p_{t-\lambda} + \tilde{r}_{t+1}, p_{t-\lambda+1}, \dots, p_{t-1})$ . This equation shows the fact that the optimal ordering quantity depends on  $p_{t-\lambda}$  and  $\tilde{r}_{t+1}$  through  $x_t + r_t + p_{t-\lambda} + \tilde{r}_{t+1}$ . Continue our deduction process following this logic we would come to the conclusion that the optimal ordering quantity  $p_t^* = p_t^*(x_t + p_{t-\lambda+1} + p_{t-\lambda+2} + \dots + p_2 + \tilde{R}_{\lambda,t})$ . Thus, we complete the proof of lemma 3. Notice that  $h_t(z)$  has exactly the same structure in comparison with the objective function given in non-leadtime case except that the loss function is replaced by a multiple integral, which is also convex. The optimality of (s,S) type policy holds as long as the loss function is convex, hence the optimal policy is characterized by  $s_t$  and  $S_t$ :

**Theorem 2**

Ordering policy in leadtime case:

If  $x_t + \sum_{i=1}^{\lambda-1} p_{t-\lambda+i} + \tilde{R}_{\lambda,t} \geq s_t$ , make no order.

If  $x_t + \sum_{i=1}^{\lambda-1} p_{t-\lambda+i} + \tilde{R}_{\lambda,t} < s_t$ , then order up to  $S_t$

In period  $t$ , issue an order to replenish the inventory to  $S_t$  if the summation of initial inventory  $x_t$ , pipeline inventory  $p_{t-\lambda+1} + p_{t-\lambda+2} + \dots + p_{t-1}$  and the expected quantity of remanufacturing products within the procurement leadtime  $\tilde{R}_{\lambda,t}$  of is lower than  $s_t$ , make no order otherwise.

From the structure of optimal policy in Theorem 2, we could conclude that there indeed exists an optimal (s,S) type inventory policy for supply chain systems with remanufacturing. However, the unavailability of expected quantity of remanufacturing products within the procurement leadtime  $\tilde{R}_{\lambda,t}$  makes the theoretical optimal policy inapplicable based just on the warehouse's perception. Chapter 3 would illustrate how to implement this theory with the help of user phase characterization.

### 3. USER PHASE CHARACTERIZATION

The result in chapter 2 indicates that it is impossible to implement the theoretical optimal inventory policy simply based on in-house information of the warehouse. The lack of information with regard to remanufacturing within the procurement leadtime reveals the essence of further data mining in the user phase. Zhang and Liu [12] discuss the synergy of reliability theory in supply chain system management. User phase characterization is the approach this paper used to implement the theoretical optimal policy.

The overall off-warehouse timespan for products in the remanufacturing supply chain model as described in figure.1 is comprised of two parts: serving time in the user phase and remanufacturing time in the remanufacturing facility. Assume that the warehouse manager is of full awareness of the process time of failure products in the remanufacturing facility, which is usually true since both the warehouse and the remanufacturing facility are under the operation of the same company in most cases. On this basis, the time that products spent in the user phase is the only issue we need to account and we could assume that the remanufacturing process could be finished in negligible time  $\mu = 0$ . This means that the value of  $\tilde{R}_{\lambda,t}$  is determined only by the product's failure behavior in the user phase. Denote that the failure time of a product in the user phase to be  $\tau$  and the quantity of products serving the user phase by period  $t$  to be  $\bar{N}_t$ . Thus, the quantity of serviceable products which is also the quantity of failed products from the user phase from period  $t$  through the procurement leadtime is:

$$R(\tau, t; \lambda) = \begin{cases} \bar{N}_{t+\lambda-\tau} - \bar{N}_{t-\tau} & \lambda \leq \tau \\ \bar{N}_t - \bar{N}_{t-\tau} + \sum_{i=t}^{t+\lambda-\tau} \xi_i & \lambda > \tau \end{cases} \quad (13)$$

As we could conclude from equation (13), when the procurement leadtime is larger than the expected lifetime of a product serving the user phase, the ordering behavior of the user phase is needed in order to estimate  $\tilde{R}_{\lambda,t}$ . The expected value of  $\tilde{R}_{\lambda,t}$  is defined as:

$$\begin{aligned} \tilde{R}_{\lambda,t} &= E_{\xi} [E_{\tau} [R(\tau, t; \lambda)]] = E_{\xi} \left[ \int_0^{\infty} R(\tau, t; \lambda) \phi(\tau) d\tau \right] \\ &= \int_L^{\infty} (\bar{N}_{t+\lambda-\tau} - \bar{N}_{t-\tau}) \phi(\tau) d\tau + \int_0^{\infty} \int_0^{\tau} \left( \bar{N}_t - \bar{N}_{t-\tau} + \sum_{i=t}^{t+\lambda-\tau} \xi_i \right) \phi(\xi) \phi(\tau) d\xi d\tau \end{aligned} \quad (14)$$

For the lifetime variable  $\tau$ , we could refer to the failure log in the user phase for more insight of the average serving time in real practice. Discretize the failure time on period basis and define  $p(k)$  as the probability of a product that has a lifetime of  $k$  periods. Then we could rewrite the expected value of  $\tilde{R}_{\lambda,t}$  as:

$$\tilde{R}_{\lambda,t} = E_{\xi} [E_k [R(k, t; \lambda)]] = \sum_{k=\lambda}^{\infty} (\bar{N}_{t+\lambda-k} - \bar{N}_{t-k}) p(k) + \int_0^{\lambda} \left( \bar{N}_t - \bar{N}_{t-k} + \sum_{i=t}^{t+\lambda-k} \xi_i \right) p(k) \phi(\xi) d\xi \quad (15)$$

By now, the missing information in implementing the theoretical optimal policy is fulfilled by exploiting the failure and ordering behavior in the user phase. This methodology opens up an alternative approach for inventory management research not only for remanufacturing systems but also for inventory studies focus solely on the warehouse operation. The alignment of reliability theory in solving inventory management problems leaves space for further exploration.

#### 4. A NUMERICAL EXAMPLE

A numerical example is introduced here to illustrate the application of (s,S) type policy for supply chain systems with remanufacturing and further verify the essentiality of user phase information. In the numerical example, the demand is assumed to be Poisson distributed with the mean quantity of 20. The policy parameters  $s_t$  and  $S_t$  for each period are illustrated in Figure 2. The trigger threshold  $s_t$  is stable in comparison with the severe frustration of the order-up-to level  $S_t$ . Figure 3 illuminates the contribution of user phase information to the inventory cost reduction under different procurement leadtimes. In a 100-decision-period model, user phase information sharing could save almost half of the total inventory cost in a scenario with leadtime of 1/4 decision horizon, i.e. 25 periods. On this basis, the alignment of remanufacturing and user phase characterization is indicated as an economically efficient way for inventory cost optimization.

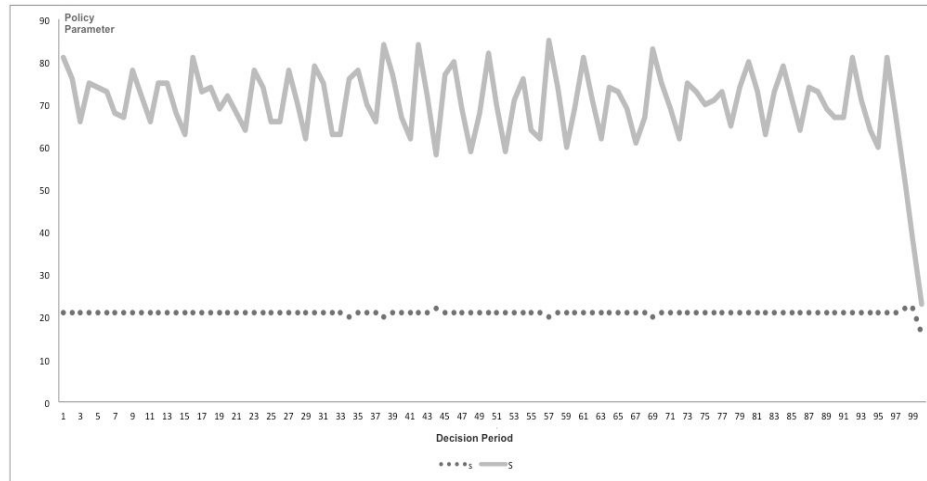


Figure 2. Decision parameter of the optimal (s,S) type policy over the planning horizon.

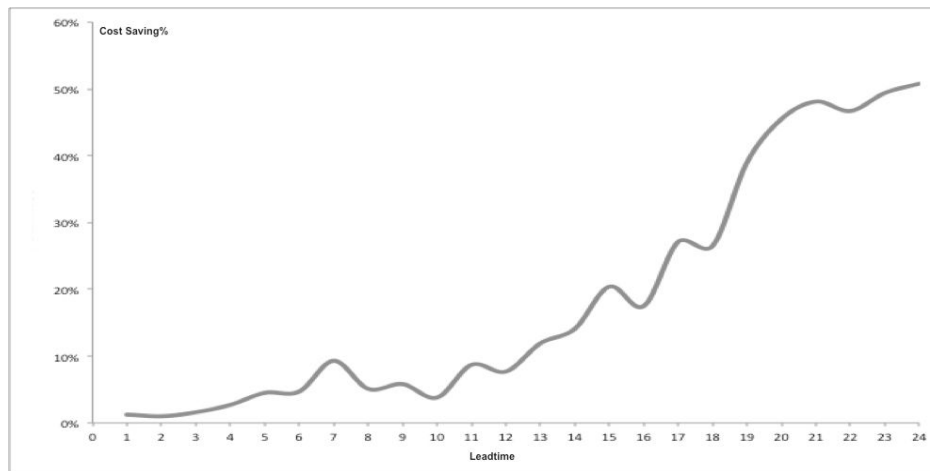


Figure 3. The contribution of user phase information sharing to inventory cost reduction under various leadtimes.

Table 1. Simulation Parameter for the numerical example.

Parameter	$N$	$K$	$h$	$\nu$	$c_p$	$\rho$
Value	100	10	0.1	5	1	0.2

## 5. CONCLUSIONS

The user phase is innovatively introduced to the conventional supply chain models and the optimality of (s,S) type policy is proved in a stochastic inventory model with procurement leadtime and fixed order set-up cost. In order to implement the theoretically optimal but practical inapplicable policy, data mining in the user phase is executed to fulfill the missing information with regard to the remanufacturing. User phase alignment is of necessity in order to acquire the information with regards to the failure behavior of servicing products. A numerical example is then used to verify its implementation and the essentiality of information sharing of the user phase.

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