A New Stochastic Simulation Optimization Methodology for Supply Chain Inventory Optimization with Imperfect Quality Items

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Abstract

This paper proposes a new stochastic simulation optimization methodology that integrates meta-heuristic with sample average approximation for supply chain inventory optimization under supply uncertainty. The supply uncertainty specifically considered is quality imperfection/deviation modelled as a discrete or continuous distribution function. In order to approximate the expected total inventory-related cost, sufficient samples of quality deviations are generated and then the corresponding sample average function is optimized by a newly developed hybrid meta-heuristic algorithm. The proposed methodology and its individual components are presented. Numerical results of a single-distributor-multiple-retailer supply chain system with each adopting the (s, S) replenishment policy indicate that the proposed methodology is capable of obtaining high quality supply chain inventory policies with percentage optimality gap within 0.01%.

1. Introduction

In most of the early literature dealing with inventory problems, the possibility of defective items is always ignored. However, defective items, and hence random yield, are common in real-world practice. These defective items will influence the on-hand inventory level, the frequency of orders and finally the service level. In order to reflect the real business environment, more and more researchers are developing inventory models in consideration of quality-related issues. Early researchers working along this line include [1] and [2], who introduce the concept of defective items in economic order quantity (EOQ) models and discuss the relationship between quality imperfection and lot sizing. Other related inventory models considering quality imperfection include those by [3-7] etc. Without exception, all of the above-mentioned analytical models focused primarily on extending the classical EOQ/EPQ models with certain set of assumptions in order to make them mathematically tractable. Khan et al. [8] present a comprehensive review of the extensions of modified EOQ/EPQ model for imperfect quality items. One of their major comments is that more research effort shall be devoted to a supply chain setting closer to practical scenarios. Research that addresses these issues may lead to modeling insights that can strengthen the coordination between all members of a supply chain.

This study is intended exactly to bridge the gap. The rapid change in modern business environment has clearly outpaced the development of mathematically tractable supply chain inventory models for real world applications. Simulation optimization, as reviewed by [9] and [10], serves as a relatively new alternative technique that has attracted attention in recent years. By combining the functionality of both a simulator and an optimizer, simulation optimization is promising in providing solutions to important practical problems previously beyond reach. To the best of our knowledge, so far no such a simulation optimization formulation has been applied to supply chain inventory management of imperfect quality products. In this study, a new stochastic simulation optimization methodology that integrates a metaheuristic algorithm with sample average approximation method is developed with the aim to find (near-)optimal inventory policies for an integrated SC model in consideration of quality imperfection in the products supplied. Meta-heuristic is chosen because it makes few or no assumptions about the problem being optimized and usually works nicely with simulation models, as shown in our earlier work in this area [11-12]. Based on [13], the sample average approximation method is modified to handle the stochastic optimization of inventory policies due to supplier quality imperfection and integrated into the metaheuristic-based simulation optimization framework.

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2. SUPPLY CHAIN INVENTORY MODEL WITH SUPPLIER QUALITY IMPERFECTION

An integrated supply chain system composed of single-distributor and multiple-retailer is considered in this study. Let entity i= 0 stand for the distributor, and entities i= 1, 2,…, M denote retailers, respectively. The retailers can be identical or non-identical. It is assumed that all parties in the supply chain adopt a periodic review inventory system, specifically the (s, S) inventory policy; but other policies can also be easily implemented. The distributor faces the possibility of receiving defective products from its supplier. The retailers, on the other hand, are faced with the uncertainty from customer demand. Our model is capable of taking any demand streams directly, without any demand distribution assumption. This study illustrates the case of a single product with the possibility of defective items in a delivered lot, based on one realization of dynamically changing demand over a finite planning horizon, T.

For each time period, t=1, 2, …,T, retailer i observes its customer demand and try to satisfy the demand with its available on-hand inventory. Any shortfall occurred at period t will be considered as lost sales to buyers with a penalty cost. Any item held at stock at period t will be charged with inventory holding cost. Next, each retailer checks its remaining inventory level and decides to order from the distributor. Two cases can be distinguished in fulfilling the demand by the distributor:

1. If the available inventory at the distributor is abundant, all retailers' orders are fully satisfied.
2. Otherwise, the orders are satisfied according to the ranking of retailers, in descending order of importance.

This study implements the ranking allocation rule but it is possible to implement other rules to adapt to different cases. In addition, it is assumed that the shortfall of each buyer's ordering quantity will be met by obtaining the shortfall from an "alternative" source but at higher cost. The distributor will be responsible for the penalty cost to this shortfall. In this way, the supply to each retailer's ordering quantity is guaranteed. The penalty cost to the distributor represents the additional cost to the distributor for obtaining items from alternative suppliers.

Next the distributor observes its inventory level and decides its ordering quantity. Note that the distributor orders from a supplier that might deliver defective items, implying that the distributor's order might not be fully satisfied. The supplier's quality deviation is included in the model following that of [14]. The non-defective items of each lot is treated as a random variable, Q_t , which follows some probability distribution P. Let $Q = \{Q_1, Q_2, ..., Q_t, ..., Q_T\}$ be one realization of the random variable over T periods and Q_t states the proportion of each order at period t that can be immediately used to fulfill demand. Therefore, for each lot ordered from the supplier, only Q_t portion can be used directly. The remaining (1-Q_t) portion failing to meet the product specification will be returned to the supplier for remanufacturing or replacement at an additional cost, and will be delivered in the following lead time period.

All orders placed to upstream are due after some constant lead time. The inventory order-up-to levels are determined accordingly to the subject inventory policy used in the system. This study adopts the widely used (s, S) policy. When the inventory position (inventory on hand plus outstanding orders) declines to or below s, some quantity of product units are ordered such that the resulting inventory position is raised to S. The decision variables thus are s_i, S_i, i = 0, 1, …,M.

The objective is to minimize the total supply chain inventory cost over the entire finite planning horizon. Given the initial system state, the total system cost G over the finite time horizon implicitly contains a random vector, Q, generated from a probability distribution P. Let X represent the set of decision variables, (s_i, S_i) policies for each entity, $i=0,…,M$. The optimization problem considered in this study is therefore of the form

$$\min_{X \in S} \{f(X) := E_P[G(X,Q)]\} \tag{1}$$

In Eq. (1), S is a finite discrete set of possible (s_i, S_i) policies, $i=0,…,M$. $E_P[G(X,Q)] = \int G(X,Q)P(dQ)$ is the corresponding expected value function. Clearly, the objective function cannot be solved analytically, but has to be measured or estimated. Given a certain demand stream and inventory policies, (s_i, S_i) values, the function can be easily computed for a given realization of Q. Also note that, the set of feasible solutions (s_i, S_i) policy for each entity in the SC, although finite, can be extremely large. The size of the feasible set grows exponentially with the number of variables and the number of periods in the planning horizon. Therefore, it is not possible to use the enumeration approach for its solution.

3. PROPOSED SIMULATION OPTIMIZATION METHODOLOGY

The proposed methodology integrates a metaheuristic-based simulation optimization framework with a sample average approximation method. The general metaheuristic-based simulation optimization framework is depicted in Fig. 1. Theoretically, any effective metaheuristic algorithm can be used here. Nevertheless, the meta-heuristic employed here is one of the most recent hybrid metaheuristics called "DE-HS-HJ" [15]. The hybrid metaheuristic comprises of two cooperative metaheusitic algorithms, i.e., differential evolution (DE) and harmony search (HS), with each enhanced by a local search (LS) method, i.e., Hooke and Jeeves (HJ) direct search. The HJ local search is applied to every new solution generated by DE and HS with a specified probability, p=0.1. This local search application strategy has been shown in [15] to be the best in terms of success rate.

Per Fig. 1, the proposed simulation optimization framework consists of two key components: a simulation model and a meta-heuristic optimizer. Simulation approximates the reality. The simulation model is used to understand what happens in the supply chain inventory system. It enables fast examination of implementing a particular decision in the "real" environment. The meta-heuristic optimizer carries out an efficient search where the successively generated inputs produce varying evaluations, not all of them improving, but which over time provide a highly efficient trajectory to the globally best solutions. The decision variable in this study is the optimal (s_i, S_i) policy for each entity of the supply chain. The solution process is initiated by inputting an initial guess of trial solutions. The population of trial solutions is sent to the simulation model. Running the simulation model generates their corresponding output performance measure, which are fed into the hybrid metaheuristic. The quality of the output guides the metaheuristic in the selection of new input solutions, based on the intelligent searching mechanism of the metaheuristic. The process is repeated until the maximum number of iterations is reached or no further improvement can be found.

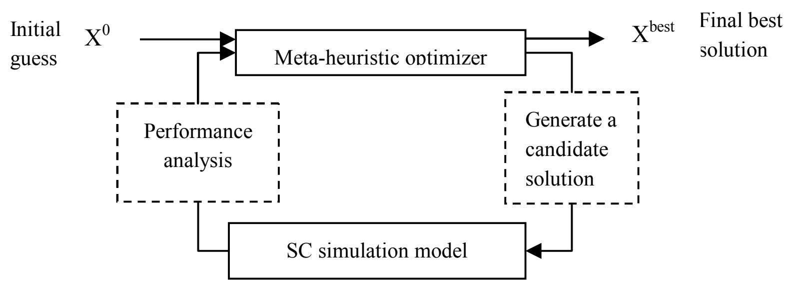

Figure 1. Metaheuristic-based simulation optimization framework to find near-optimal solutions.

In order to obtain a high quality solution for the considered stochastic optimization problem, an effective procedure is needed to select the best, if not really optimal, solution with high probability. To this end, the work of [13], in which they originally developed an approximation method based on the Monte Carlo simulation approach, is adapted and integrated into the metaheuristic-based simulation optimization framework. The basic idea is that a set of random samples is generated and the expected value function is approximated by the corresponding sample average function. The obtained sample average problem is optimized by the hybrid meta-heuristic and the procedure is repeated until the stopping criterion is satisfied.

Let $Q^1, Q^2, ..., Q^j, ..., Q^N$ be a set of independent and identically distributed (i.i.d.) random samples of N realizations of the random vector Q. Equation (1) is approximately solved by

$$\min_{X \in S} \hat{f}_N(X) \tag{2}$$

and

$$\hat{f}_N(X) := \frac{1}{N} \sum_{j=1}^{N} G(X, Q^j) \tag{3}$$

Note that $E[\hat{f}_N(X)] = f(X)$. Unless the random vector Q has a small number of possible realizations (scenarios), it is usually impossible to solve the problem exactly. The proposed methodology is primarily concerned with solving the sample average approximating problem by meta-heuristic and bounding the solution quality using the so-called optimality gap.

Let z^* and $\hat{z}_N$ denote the optimal values of the respective problems,

$$z^* := \min_{X \in S} f(X) \text{ and } \hat{z}_N := \min_{X \in S} \hat{f}_N(X) \tag{4}$$

The idea is to use the metaheuristic-based simulation optimization framework to approximate $\hat{z}_N$. The corresponding pseudo-code of the metaheuristic-based simulation optimization framework is given as follows:

---------------------------Algorithm 1: Metaheuristic-based simulation optimization algorithm-------------------------------

Specify meta-heuristic-related parameters, number of maximum evaluations, *Maxnfe, &* size of population*, NP*.
(1) Randomly generate the initial population of solutions, X^0.
(2) Draw *N* samples of quality realizations.
(3) Evaluation the initial set of solutions through the simulation model and determine the objective value of the sample average problem of Eq. (3) for each solution. Let current number of evaluation, *nfe*=*NP*. Determine the best objective value, $f^{best,}$ and the best solution, X^{best}, of the initial population.
(4) While *nfe*<*Maxnfe* or the improvement of f^{best}>0.1 , do
    For each target solution in the population (=1, 2, …, *NP*)
        Generate a trial vector according to the DE algorithm;
        Construct a new solution according to the HS algorithm;
        If the new solutions are randomly selected (with a specified percentage, *p*)
            Apply the HJ local search to improve the solution to its nearest local optimum under the *N* quality samples and increment *nfe* accordingly;
        Else
            Evaluate the new solutions found by both algorithms through the simulation model and determine their corresponding objective values of the sample average problem of Eq. (3);
            Increment *nfe* by two, i.e., *nfe*=*nfe*+2.
        End if
        Replace the target solution with the new trail solution for the DE algorithm and HS algorithm, respectively, if there is an improvement.
        Update the best solution among the whole population, X^{best}, found so far
    End for
    End while
(5) Output X^{best}

To obtain more than one replicate, the simulation optimization framework is run *L* times such that *L* is statistically sufficient (>30). In each run *l*, *l* = 1, …, *L*, a near-optimal solution $\hat{X}_N^l$ is obtained. Note that meta-heuristic cannot guarantee finding the true optimum 100%. Therefore, all candidate solutions $\hat{X} = \{\hat{X}_N^1, ..., \hat{X}_N^l, ..., \hat{X}_N^L\}$ found by Algorithm 1 are first subjected to a screening procedure to get rid of significantly inferior solutions. The screening procedure is based on Hsu's multiple comparisons with the best procedure [16]. To determine whether the resultant subset size is large enough, the % difference in overall average expected cost is tracked and compared with a specified cutoff value. Let the current solution number to be *C*, the "% difference" values is calculated as

$$\{[\frac{1}{C}\sum_{k=1}^{C}\hat{f}_N(\hat{X}'^k_N)]\Big/[\frac{1}{C-1}\sum_{k=1}^{C-1}\hat{f}_N(\hat{X}'^k_N)]-1\}\cdot 100\% \tag{5}$$

After the screening procedure and the subset size is determined large enough, the subset $\hat{X}' = \{\hat{X}'^1_N, ..., \hat{X}'^k_N, ..., \hat{X}'^K_N\}$, $(K \le L)$ solutions that are not significantly inferior to the "best" is chosen to estimate the true optimum z^* as

$$\bar{z}_N^K = \frac{1}{K}\sum_{k=1}^{K}\hat{f}_N(\hat{X}'^k_N) \tag{6}$$

where $\hat{X}'^k_N$ denotes the *k*th solution in the subset $\hat{X}'$. Note that $E[\bar{z}_N^K] = E[\hat{z}_N]$, and hence the estimator $\bar{z}_N^K$ has the same negative bias as $\hat{z}_N$. The subset solution $\hat{X}'$ found by the hybrid meta-heuristic probably contains true (near-)optimal solution for the stochastic supply chain inventory problem. For each candidate solution $\hat{X}'^k_N$, the corresponding expected cost of operating the inventory system, $f(\hat{X}'^k_N)$, is computed via the sample average estimator

$$\hat{f}_{N'}(\hat{X}'^k_N) = \frac{1}{N'}\sum_{j=1}^{N'}G(\hat{X}'^k_N, Q^j) \tag{7}$$

In Eq. (7), it is suggested to use a larger sample size of quality realizations such that *N'*>*N* to obtain an accurate estimate $\hat{f}_{N'}(\hat{X}'^k_N)$ to the objective value $f(\hat{X}'^k_N)$ of solution $\hat{X}'^k_N$. It has been shown in [13] that it is an unbiased estimator of the true cost of the solution $\hat{X}'^{k}_{N}$, i.e.,

$$E[\hat{f}_{N'}(\hat{X}'^{k}_{N})] \geq z^* \tag{8}$$

and $\hat{f}_{N'}(\hat{X}'^{k}_{N}) \to f(\hat{X}'^{k}_{N})$ convergence with probability one (w.p.1) as $N' \to \infty$. A measure of the accuracy of $\hat{f}_{N'}(\hat{X}'^{k}_{N})$ or $f(\hat{X}'^{k}_{N})$ is given by the corresponding sample variance, $S^2_{N'}(\hat{X}'^{k}_{N})/N'$, which can be calculated from the same sample of size N'.

Of course, the chosen N' involves a trade-off between computational effort and accuracy measured by $S^2_{N'}(\hat{X}'^{k}_{N})/N'$. Typically, estimating the value of function (7) for a candidate solution $\hat{X}'^{k}_{N}$ by the sample average estimator $\hat{f}_{N'}(\hat{X}'^{k}_{N})$ requires much less computational effort than trying to minimize the sample average approximation problem (function 2). Thus, it is feasible to use a significantly large sample size N' to estimate the true cost of operating the supply chain inventory system with the solution $\hat{X}'^{k}_{N}$. In this study, we select to follow [17] and choose the second stage sample size N' as follows:

$$N' = \arg\max_{i=1,\ldots,K}\{N'_k\} \tag{9}$$

$$N'_k = \max\{N, \left\lceil (\frac{h}{\delta})^2 S^2(\hat{X}'^{k}_{N}) \right\rceil\}\} \tag{10}$$

where $\lceil y \rceil$ is the smallest integer that is greater than or equals to y; N'_k is the lower bound of the sample size needed for evaluating solution $\hat{X}'^{k}_{N}$; $S^2(\hat{X}'^{k}_{N})$ is the corresponding sample variance of solution $\hat{X}'^{k}_{N}$; δ is the indifference zone determined by the decision maker; h can be obtained from a table given in [18].

To determine the performance bound of each solution found by our simulation optimization framework, the optimality gap $f(\hat{X}'^{k}_{N}) - z^*$ for a candidate solution $\hat{X}'^{k}_{N}$ is computed. Unfortunately, the very reason for us to develop the new framework implies that both terms of the optimality gap are hard to compute exactly. Recall that $E[\hat{z}_N] \leq z^* \leq E[\hat{f}_{N'}(\hat{X}'^{k}_{N})]$ and $E[\bar{z}^K_N] = E[\hat{z}_N]$. Hence, the optimality gap $f(\hat{X}'^{k}_{N}) - z^*$ is estimated by $\hat{f}_{N'}(\hat{X}'^{k}_{N}) - \bar{z}^K_N$, at the solution $\hat{X}'^{k}_{N}$ and it can be obtained that

$$E[\hat{f}_{N'}(\hat{X}'^{k}_{N}) - \bar{z}^K_N] = f(\hat{X}'^{k}_{N}) - E[\hat{z}_N] \geq f(\hat{X}'^{k}_{N}) - z^* \tag{11}$$

It follows that on average the above estimator overestimates the optimality gap $f(\hat{X}'^{k}_{N}) - z^*$. The bias $z^* - E[\hat{z}_N]$ is monotonically decreasing with the sample size N. If the K quality realizations of size N and the evaluation samples of size N' are independent. The variance of the optimality gap estimator $\hat{f}_{N'}(\hat{X}'^{k}_{N}) - \bar{z}^K_N$ can be estimated by the sum of $S^2_{N'}(\hat{X}'^{k}_{N})/N'$ and S^2_K/K .Specifically, the variance of $\hat{f}_{N'}(\hat{X}'^{k}_{N})$ is estimated by

$$S^2_{N'}(\hat{X}'^{k}_{N})/N' = \frac{1}{N'(N'-1)}\sum_{j=1}^{N'}(G(\hat{X}'^{k}_{N}, Q^j) - \hat{f}_{N'}(\hat{X}'^{k}_{N}))^2 \tag{12}$$

The variance of $\bar{z}^K_N$ is estimated by

$$S^2_K / K = \frac{1}{K(K-1)}\sum_{k=1}^{K}(\hat{X}'^{k}_{N} - \bar{z}^K_N)^2 \tag{13}$$

Following the Central Limit Theorem, for sufficiently large N and N', the accuracy of an optimality gap estimator can be taken into account by multiplying a z_α of its estimated standard deviation to the gap estimator. Note that $z_\alpha = \Phi^{-1}(1-\alpha)$, $\Phi(z)$ is the cumulative distribution function of the standard normal distribution. The approximate (1-2 α)-level confidence interval (CI) for the optimality gap for a given near-optimal solution $\hat{X}'^{k}_{N}$ found by the hybrid meta-heuristic is given by

$$[0, \hat{f}_{N'}(\hat{X}'^{k}_{N}) - \bar{z}^K_N + z_\alpha(\frac{S^2_{N'}(\hat{X}'^{k}_{N})}{N'} + \frac{S^2_K}{K})^{1/2}] \tag{14}$$

Due to the possibility of sampling error, $\hat{f}_{N'}(\hat{X}'^{k}_{N}) < \bar{z}^{K}_{N}$ might actually be observed. Hence, it is recommended to use the following more conservative confidence interval

$$[0,[\hat{f}_{N'}(\hat{X}'^{k}_{N}) - \bar{z}^{K}_{N}]^{+} + z_{\alpha}(\frac{S^{2}_{N'}(\hat{X}'^{k}_{N})}{N'} + \frac{S^{2}_{K}}{K})^{1/2}] \qquad (15)$$

where $[x]^{+} = \max\{x,0\}$

Select the solution that has the smallest optimality gap and label it as $\hat{X}*$. In other words, the proposed methodology guarantees with probability at least equal to $(1\text{-}2\alpha)$ that the chosen solution has the best objective value $f(\hat{X}*)$ over all candidate solutions $\hat{X}'$ and the estimated error of its optimality gap is at most equal to the upper endpoint of Eq. (15). Algorithm 2 summarizes the proposed methodology.

-----------------------Algorithm 2: The proposed methodology to obtain a high quality solution---------------------------

(1) Choose initial sample size *N* for quality imperfection realizations, a decision rule for determining the number of meta-heuristic runs *L* with the sample size *N*

(2) For *l=1,…,L*, do

  1.1. Generate a set of quality samples of size *N* and try to solve the sample average approximation problem (function 2) using Algorithm 1 and record the objective value $\hat{z}^{l}_{N}$ and suboptimal solution $\hat{X}^{l}_{N}$.

  1.2. Perform the screening procedure of Hsu's MCB to get rid of solutions that are significantly inferior to the best solution and obtain the subset $\hat{X}'$

  1.3. Test whether the subset size is large enough by tracking whether % difference in overall average expected cost is less than a specified cutoff value. Increase *L* if not.

End for

(3) Using the subset $\hat{X}' = \{\hat{X}'^{1}_{N},...,\hat{X}'^{k}_{N},...,\hat{X}'^{K}_{N}\}$ to calculate $\bar{z}^{K}_{N}$ (function 6)

(4) Choose a sufficiently larger sample size $N' > N$

For each candidate solution $\hat{X}'^{k}_{N}$, *k=1,…,K*, do

  Estimate $f(\hat{X}'^{k}_{N})$ (the expected cost of operating the inventory system with the solution $\hat{X}'^{k}_{N}$) via the sample average estimator $\hat{f}_{N'}(\hat{X}'^{k}_{N})$ (function 7)

End for

(5) Estimate the optimality gap and construct the confidence interval by Eq. (15)

(6) Choose the solution among all candidate solutions $\hat{X}'$ which has the smallest optimality gap as the best solution, $\hat{X}*$.

4. NUMERICAL EXAMPLE

As an illustration, a single-item SC inventory system with single distributor and three retailers is considered. One salient feature of our supply chain inventory model is that any demand stream can be handled. Those demand series do not have to follow any particular distribution/assumption. However, no demand uncertainty is considered in this study. The distributor must confront the quality uncertainty from its main supplier and fill up the shortfall caused by those defects from another more expensive supplier. The quality level of delivered lots is assumed to follow a simple discrete distribution with three possible events: worst=0.8, most likely=0.9 and best=1. Quality level of 0.8 in the worst event means that 80% of the supplied items conformed to the specification and can be used to fulfill customer demand directly. The probability of these three events is assumed to be *P*(0.8)=0.1, *P*(0.9)=0.8, *P*(1)=0.1. The aim is to optimize the (s_i, S_i) policy for each entity in the SC so that the total expected inventory-related cost over the finite planning horizon is minimized. Because the quality uncertainty exists in the SC system, the optimization problem is of the form shown in Eq. (1). The proposed methodology is applied to find the set of candidate solutions by using Algorithm 1 and to select a high quality near-optimal solution by following Algorithm 2.

The parameter settings were chosen based on their good performance from our previous experience. The maximal number of function evaluations, *MaxFE*=50000, was used as the stopping criterion. The on-hand inventory in the SC was initialized to be 200 for retailers and 400 for distributor at the beginning of period 0. Inventory-related parameters such as lead time, ordering cost per order, holding cost per unit item and unit time, lost sale cost per unit item, remanufacturing cost per unit item, were fixed to be 2 periods, \$100, \$2, \$20 and \$10, respectively. The planning horizon was assumed one year with 52 weekly demand data and all three retailers had the same demand stream generated from a Gamma distribution with mean and standard deviation equals to 49 and 4.9, respectively. The random quality vector was realized 400 times (*N=400*) and consistently used in all 50 replicates of meta-heuristic runs (*L=50*). The lower and upper bounds of possible s_i values for each supply chain entity were set as the minimum weekly demand and the maximum lead-time demand, respectively; the lower and upper bounds of possible S_i values for each supply chain entity were set as the minimum weekly demand and the doubled maximum lead-time demand, respectively. There were totally 8 variables to be optimized (4 entities in the SC system and each has 2 values to be optimized). All programs were coded in Matlab and all executions were made on a HP Pavilion a4317c with AMD Athlon ™ II × 2@ 2.70 GHz.

At the end of Step 2 of Algorithm 2, 50 candidate solutions $\hat{X}$ and their estimated expected costs for all 400 realizations of the quality vector were obtained. The screening procedure was subsequently applied to get rid of solutions that are significantly inferior to the best solution. The Hsu's MCB test result indicates that 15 solutions among the total 50 are significantly inferior to the best and shall be removed from further analyses. The remaining 35 solutions constitute the subset of *X′*. Whether the subset of *X′* is large enough is determined by tracking the percentage difference of the average expected cost according to Eq. (5). The % difference greatly fluctuates when the subset size of indifferent solutions is small and gradually stabilizes as the subset size increases. Using 0.0008% as the cutoff, a subset size of 35 solutions is determined large enough, meaning that additional replications are not likely to be justified. Based on these 35 solutions, $\bar{z}_N^K$ was computed (*N*=400 and *K*=35 in this case) to estimate the true *z** of the problem and the associated variance was computed by S_K^2.

Each of these 35 solutions was further evaluated using a much larger sample size N' to estimate the expected cost of operating the supply chain inventory system. N'=100,000 were tested. Note this size is sufficiently larger than the bound provided by [18]. Tables 1 record the corresponding results. It can be observed that the best solution is #32: *s* = [78 77 89 252]; *S*=[152 152 152 313], which means that the near-optimal (s_i, S_i) policy for the three retailers and the distributor are (78, 152), (77, 152), (89, 152) and (252, 313), respectively. Besides Solution 32, actually a large portion of all 35 solutions (around 43%) is expected to perform well. Their optimality gaps are all very small. Note that the optimality gaps are absolute values which are almost negligible relative to the system expected cost. For those high quality solutions, the percentage optimality gaps are all within 0.01%. Generally speaking, increasing the confidence level tends to expand the CI a little bit, which means the margin of error will be increased in order to increase the confidence level.

Last but not least, note that the upper boundary of the optimality gap estimator (Eq. 15) can be divided into components as follows:

$$(\hat{f}_{N'}(\hat{X}^*) - f(\hat{X}^*)) + (f(\hat{X}^*) - z^*) + (z^* - \bar{z}_N^K) + z_\alpha\left(\frac{S_{N'}^2(\hat{X}^*)}{N'} + \frac{S_K^2}{K}\right)^{1/2} \tag{16}$$

The first term of the above expression has an expected value zero if $N' \to \infty$; The second term is the true optimality gap; the third term has a positive expected value that is decreasing as $N \to \infty$; the last term is the measure of accuracy, which decreases with increased number of replications *K* and the sample size N'. Even with an optimal solution $\hat{X}^*$, i.e., $f(\hat{X}^*) - z^* = 0$, the value of other three terms can be large if *K, N* and N' are small, Therefore, it is recommended to use a sufficient large number of *K, N* and N' to achieve the desired level of accuracy.

Table 1.Test results of candidate solution based on *N′*=100,000 quality scenarios.

NO.	candidate solution	estimated expected cost	estimated variance	Optimality gap	
				90% CI	95% CI
1	s=[93 75 86 250] S=[154 151 152 311]	65613.21558	909645.376	28.76900047	29.89766785
2	s=[70 74 78 195] S=[154 150 152 311]	65617.08094	899836.1461	32.61179412	33.73613838
⋮	⋮	⋮	⋮	⋮	⋮
32	**s=[78 77 89 252] S=[152 152 152 313]**	**65588.62458**	**859016.8015**	**5.77414413**	**6.880317095**
33	s=[88 85 78 222] S=[152 152 152 311]	65581.48664	906131.6026	5.883489764	7.010610459
34	s=[78 75 78 222] S=[152 152 152 312]	65579.93974	878485.5057	5.819576843	6.934453517
35	s=[86 87 87 199] S=[152 152 152 312]	65579.93974	878485.5057	5.819576843	6.934453517

5. SUMMARY

This paper has presented a new stochastic simulation optimization methodology that integrates a metaheuristic-based simulation optimization framework with a modified sample average approximation method for optimizing integrated supply chain inventory systems with supplier quality imperfection. Using the proposed methodology, the (near-)optimal inventory policies for each entity within the supply chain can be estimated and the optimality gap can be quantified. The proposed methodology was illustrated with an example to find the optimal (*s, S*) policies for an integrated supply chain system composed of a single-distributor multiple-retailer system with the supplier product quality modeled as a three-point discrete distribution function. The results shows that the proposed procedure is capable of finding high quality solutions with the percentage optimality gaps are all within 0.01%.

A possible topic for future research is developing approaches that offer better tradeoff between problem solvability, solution accuracy and computational expenses.

REFERENCES

[1] Porteus, E.L.: "Optimal lot sizing, process quality improvement and setup cost reduction", Operations Research, 34 (1), 137–144, 1986.

[2] Rosenblatt, M.J., and Lee, H.L.: "Economic production cycles with imperfect production processes", IIE Transactions, 18(1), 48–55, 1986.

[3] Balkhi, Z.T.:. "An optimal solution of a general lot size inventory model with deteriorated and imperfect products, taking into account inflation and time value of money", International Journal of Systems Science, 35(2), 87–96, 2004.

[4] Papachristos, S., & Konstantaras, I.: "Economic ordering quantity models for items with imperfect quality", Int. J. Production Economics, 100(1), 148–154, 2006.

[5] Wee, H.M., Yu, J., & Chen, M.C.: "Optimal inventory model for items with imperfect quality and shortage backordering", OMEGA, 35 (1), 7–11, 2007.

[6] Mondal, B., Bhunia, A.K., and Maiti, M.: "Inventory models for defective items incorporating marketing decisions with variable production cost", Applied Mathematical Modeling, 33(6), 2845–2852, 2009.

[7] Barzoki, M.R., Jahanbazi, M., and Bijari, M.: "Effects of imperfect products on lot sizing with work in process inventory", Applied Mathematics and Computation, 217(21), 8328–8336, 2011.

[8] Khan, M., Jaber, M.Y., Guiffrida, A.L., and Zolfaghari, S.: "A review of the extensions of a modified EOQ model for imperfect quality items", International Journal of Production Economics, 132(1), 1–12, 2011.

[9] Fu, M.C.: "Optimization for simulation: Theory vs. practice", INFORMS Journal on Computing, 14(3), 192–215, 2002.

[10] Tekin, E., Sabuncuoglu, I.: "Simulation optimization: A comprehensive review on theory and applications", IIE Transactions, 36, 1067–1081, 2004.

[11] Liao, T.W., and Chang, P.C.: "Impacts of forecast, inventory policy, and lead time on supply chain inventory cost – A numerical study", Int. J. Production Economics, 128(2), 527–537, 2010.

[12] Duan, Q., and Liao, T.W.: "Optimization of replenishment policies for decentralized and centralized capacitated supply chains under various demands", International Journal of Production Economics, 142(1), 194–204, 2013.

[13] Kleywegt, A.J., Shapiro, A., and Homem-De-Mello, T.: "The sample average approximation method for stochastic discrete optimization", SIAM journal on optimization, 12(2), 479–502, 2001.

[14] Mahnam, M., Yadollahpour, M.R., Famil-Dardashti, V., and Hejazi, S.R.: "Supply chain modeling in uncertain environment with bi-objective approach", Computers & Industrial Engineering, 56(4), 1535–1544, 2009.

[15] Duan, Q., Liao, T.W., and Yi, H.Z.: "A comparative study of different local search application strategies in hybrid metaheuristics", Applied Soft Computing, 13(3), 1464–1477, 2013.

[16] Hsu, J.C: "Simultaneous confidence intervals for all distances from the 'best'", Annals of Statistics, 9(5), 1026–1034, 1981.

[17] Rinott, Y: "On two-stage selection procedures and related probability inequalities", Communications in Statistics, A7(8), 799–811, 1978.

[18] Wilcox, R.R: "A table for Rinott's Selection Procedure", J. Quality Technology, 16(2), 97–100, 1984.