Comparative Analysis Between Small Displacement Torsor and Model of Indeterminate Applied On Generated Solution of Reconfigurable Manufacturing System

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ABSTRACT

Reconfigurable manufacturing system (RMS) is the recent addition in the field of manufacturing system. Different approaches deal with the generation of design solutions of RMS. Keeping in view quality as a key performance indicator there is a necessity to evaluate the generated solutions of RMS. In this work a comparative study between the two methods for evaluation of tolerances is performed. First using an algorithmic approach a three dimensional analysis of generated solutions of RMS is carried out. Secondly the model of the indeterminate is used for the tolerance analysis. In both methods the approach used among the existing ones for the tolerance representation and evaluation is the small displacement torsors (SDT). In order to represent the machining process plans the modified method of graph is chosen. In the algorithmic approach the torsor equations are obtained between the interacting surfaces. Each geometric error is represented as a torsor which are then accumulated. Solutions are classified according to the tolerance values of the parameters. In the second method the gaps and defects of the surfaces are first identified and then are written in the form of torsors. The compatibility equations are obtained by resolving the loop equations. These equations are analyzed to obtain the sources of error and eventually part tolerances are calculated. The above methodologies have wide applications in the generative approach for process planning of RMS. They provide a direct link between the sources of error and part requirements. The said methodology acts as a feedback system for the capability of the system.

1. INTRODUCTION

Reconfigurable manufacturing systems (RMS) come into existence because of the demanding needs of the market in a frequent, efficient and cost effective manner. It fulfills the unpredictable market changes which are due to the introduction of new products and constantly varying demands of different products. It also has the capability to produce high quality products at low cost [1].

The concept of co-evolution is used for the generation of kinematic configurations and the process plans. Conventional approaches are no more applicable for the generation of structural configurations and process plans. In co-evolution, the process plans and kinematic configurations are generated simultaneously instead of the conventional approaches where either of them is generated first and then passed to the next step. Product quality is the main objective of this approach [2] [3]. The method given by El Maraghy [4] is used in this work for the generation of machine configurations. In this method the machine configurations and kinematic configurations changes as the product features changes. Cutting tool charts, sequence tables and precedence relationship matrix are the input to this model and process plans are the output, represented in the form of a hierarchical tree structure [4] [5].

Small displacement torsors are used in order to measure the errors in a process plan. They are of four types [2] [6] [7].

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- *Error torsor* represents the displacement between a theoretical nominal surface and the position of the real surface. This torsor only depends upon the topology of the surface. This torsor represents the dimensional errors of machined surfaces
- *Deviation torsor* represents the deviation of difference in position between two surfaces of the same work piece
- *Connection or Play* torsor represents the positioning error between two surfaces of two solids. This torsor represents contact error between part holder and part surfaces and also between machining operations and machined surfaces
- *Global torsor* represents the deviations of position of a solid with respect its nominal position. This toror is used to give deviations in machine tool and tool positions

The types of torsor that exist between different elements in a machining setup are indicated in figure 1:

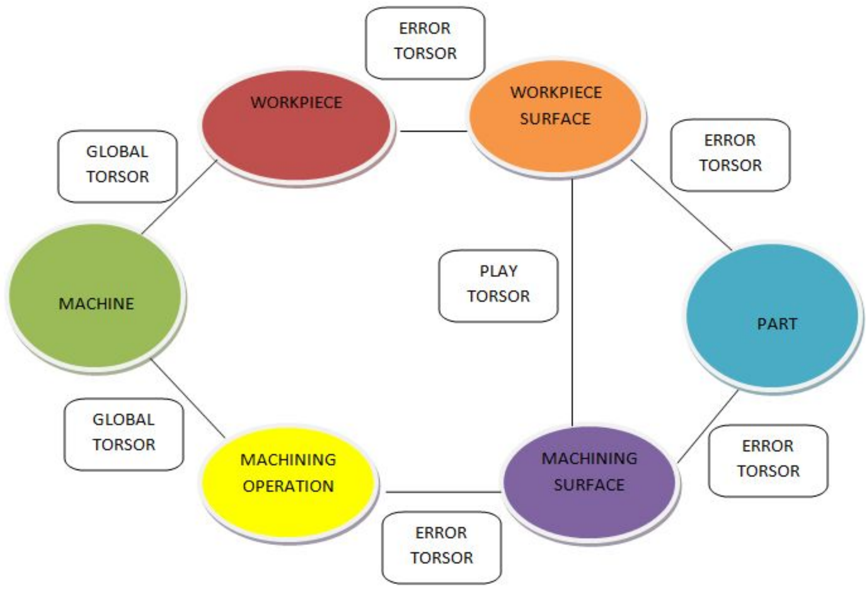

Figure 1. Torsor setup in a machining phase.

In this article tolerance analysis is done by two different methods. Deviations in both the methods are written in the form of small displacement torsors (SDT). Tolerance analysis and synthesis of machining tolerances is performed on the basis of the geometrical errors represented in the form of SDT [8] [9]. 3D tolerance analysis [6] [7] is performed.

2. METHODOLOGY/ TOLERANCE MODELS

2.1. ALGORITHMIC APPROACH

An iterative approach is used in this article for the tolerance analysis of generated solutions of RMS. This approach gives us the absolute value of tolerances which reflexes the capability of the manufacturing system. It starts with the initialization of one of the generated process plans and goes on till all process plans are analyzed. Torsor chains are generated and the torsor equations are obtained. In these equations each element represents a torsor whose value is determined in the next step. After that the data values are plugged into the torsors and the deviations are obtained. Basing on the value of tolerance we can decide whether the process plan is feasible or not. The approach used in this article is shown in figure 2:

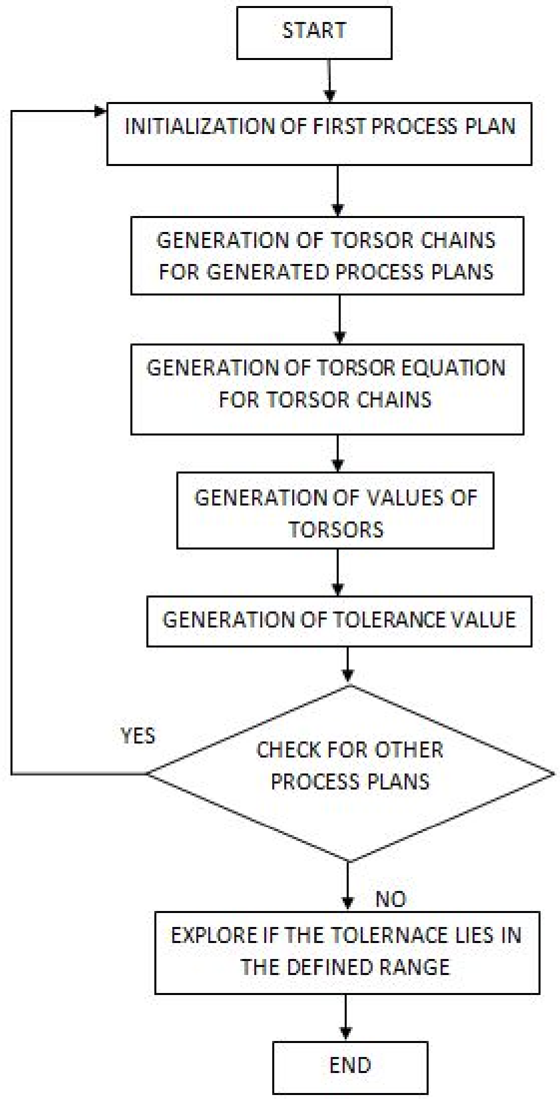

Figure 2. Flowchart.

2.2. MODEL OF INDETERMINATES

The study of geometric behavior of successive actual states of the part during the manufacturing process requires relations to be written between the functional conditions, the geometric defects of the parts and the gap in the links. These relations can be obtained from the model of indeterminate which is a generalization of the ΔL method. This method is based on the following steps:

Step 1: Deviation torsors for the calculated surfaces

Step 2: Gap torsors for pairs of relative positioning calculated surfaces

Step 3: Geometric loop closing equation of the calculated surfaces

Step 4: Compatibility relations

Step 5: Resolve and obtain system of equations

From the above system of equations the following results can be obtained:

- Indeterminate values as function of differences in gap and deviation torsors
- Degrees of freedom of the system or mechanism from the indeterminate variables which are not determined by the resolving the system of equations
- Chains of deviations in 3d from the compatibility relations between the gap and geometric defects of the surfaces if the system of equation is over-constrained [10] [11]

3. APPLICATION

The part CAI (cover indeterminate shaft) is selected as a part on which both the methods are applied and the results are generated. The machining features are indicated in figure 3. The single post generated solutions of the mentioned part are used for the tolerance analysis. Single post generated solution and the liaisons between the interacting surfaces are indicated in figures 4 & 5 respectively. The application is same as used by Arsalan and Aamer [2] during their work in which the algorithmic methodology was proposed.

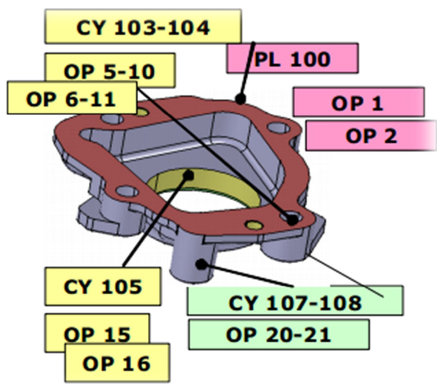

Figure 3. Part cover indeterminate shaft [6].

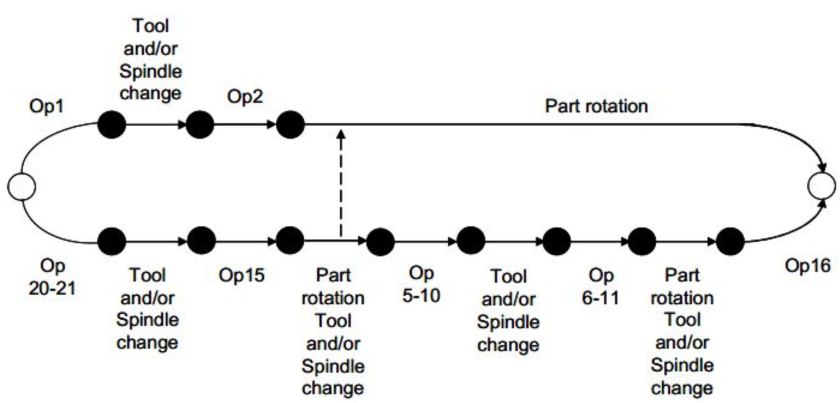

Figure 4. Single post generated solution [6].

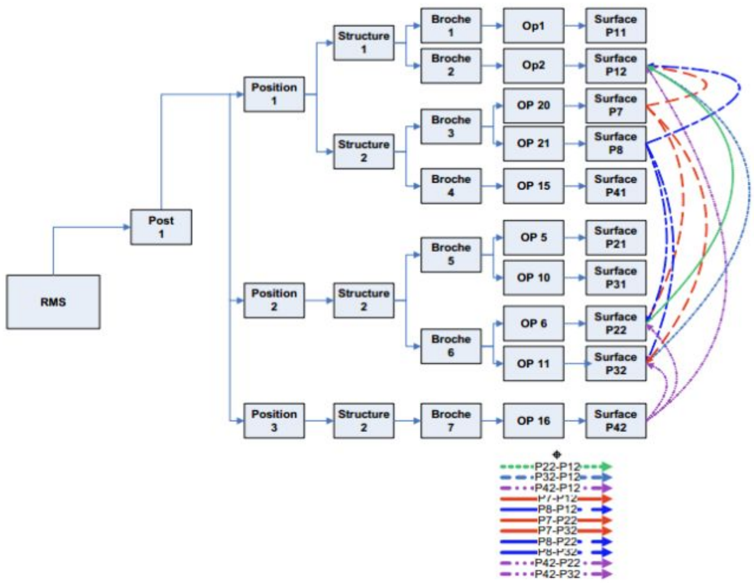

Figure 5. Liaisons between the interacting surfaces [6].

4. RESULTS & ANALYSIS

The generated solution for single post shows 11 possible liaisons between the interacting surfaces as shown in figure 5. First of all 3D analysis of the part CAI is performed using the algorithmic method. The interaction between the surfaces P32-P12 is kept under consideration.

$$T_{P32-P12} = \Delta T_{Tooling\ 12} + \Delta T_{Broche\ 2} + \Delta T_{Structure\ 1} + \Delta T_{Position\ 1} + \Delta T_{Position\ 2} + \Delta T_{Structure\ 2} + \Delta T_{Broche\ 6} + \Delta T_{Tooling\ 32} \quad (1)$$

$$T_{P32-P12} = \begin{bmatrix} \Delta\alpha_{T12} & \Delta x_{T12} \\ \Delta\beta_{T12} & \Delta y_{T12} \\ \Delta\gamma_{T12} & \Delta z_{T12} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B2} & \Delta x_{B2} \\ \Delta\beta_{B2} & \Delta y_{B2} \\ \Delta\gamma_{B2} & \Delta z_{B2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{S1} & \Delta x_{S1} \\ \Delta\beta_{S1} & \Delta y_{S1} \\ \Delta\gamma_{S1} & \Delta z_{S1} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P1} & \Delta x_{P1} \\ \Delta\beta_{P1} & \Delta y_{P1} \\ \Delta\gamma_{P1} & \Delta z_{P1} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{P2} & \Delta x_{P2} \\ \Delta\beta_{P2} & \Delta y_{P2} \\ \Delta\gamma_{P2} & \Delta z_{P2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{S2} & \Delta x_{S2} \\ \Delta\beta_{S2} & \Delta y_{S2} \\ \Delta\gamma_{S2} & \Delta z_{S2} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B6} & \Delta x_{B6} \\ \Delta\beta_{B6} & \Delta y_{B6} \\ \Delta\gamma_{B6} & \Delta z_{B6} \end{bmatrix} + \begin{bmatrix} \Delta\alpha_{B6} & \Delta x_{B6} \\ \Delta\beta_{B6} & \Delta y_{B6} \\ \Delta\gamma_{B6} & \Delta z_{B6} \end{bmatrix} \quad (2)$$

The data values basing on the experimental data and the input from the experienced machine operators are as shown in table 1. The associated error/ deviation in different activities related to tool, spindle, structure, position and part change activities in part manufacturing process are catered through these values.

Table 1. Data Values.

Values are in mm/ deg for translations/ rotation						
Elements	**Δx**	**Δy**	**Δz**	**Δα**	**Δβ**	**Δγ**
Tool	.002	.002	.002	.08	.08	.08
Spindle	.004	.004	.004	.08	.08	.08
Structure	.003	.003	.003	.08	.08	.08
Position	.004	.004	.004	.08	.08	.08
Post	.006	.006	.006	.08	.08	.08

Using the above values and incorporating the Varignon relationship the result comes out to be [12] [13]:

$$T_{P32-P12} = \begin{bmatrix} 0.64 & 0.026 \\ 0.64 & 0.026 \\ 0.64 & 0.026 \end{bmatrix} \tag{3}$$

From the equation no. 3 we obtained the variations in three dimensions. First column in the torsor in equation no. 3 gives the deviation of rotations and the second gives the deviation of translations in the respective x, y & z axis.

Now the model of indeterminate is applied on the same interacting surfaces P_{32}-P_{12}. In this method the steps as mentioned above are followed. This method gives a system of equations that leads to part tolerances. If the system of equation is indeterminate then few assumptions and conditions are applied to obtain the end result. In case of unconstrained system each relation will lead a chain to a loop which is closed by a functional condition. In over-constrained system specific relations will express condition of compatibility between gaps and defects.

STEP 1: Deviation torsors of the respective calculated surfaces P_{32} & P_{12} are:

$$E_{P32} = \begin{bmatrix} a_{32} & u_{32} \\ b_{32} & v_{32} \\ c_{32} & w_{32} \end{bmatrix} \tag{4}$$

$$E_{P12} = \begin{bmatrix} a_{12} & u_{12} \\ b_{12} & v_{12} \\ c_{12} & w_{12} \end{bmatrix} \tag{5}$$

STEP 2: Gap torsors between the interacting surfaces is:

$$T_{(P32/P12)} = \begin{bmatrix} J\,(r_x, P_{32}, P_{12}) & J\,(t_x, P_{32}, P_{12}) \\ J\,(r_y, P_{32}, P_{12}) & J\,(t_y, P_{32}, P_{12}) \\ J\,(r_z, P_{32}, P_{12}) & J\,(t_z, P_{32}, P_{12}) \end{bmatrix} \tag{6}$$

Where J represent the components of the gap torsor.

STEP 3: Loop equations

$$T\,(P_{32}, P_{12}) = E\,(P_{32}/P_i) + D\,(P_i/R) - D\,(P_j/R) - E\,(P_{12}/P_j) \tag{7}$$

$$T\,(P_{22}, P_{11}) = E\,(P_{22}/P_i) + D\,(P_i/R) - D\,(P_j/R) - E\,(P_{11}/P_j) \tag{8}$$

$$T\,(PL_{100}, PL_{101}) = E\,(PL_{100}/P_i) + D\,(P_i/R) - D\,(P_j/R) - E\,(PL_{101}/P_j) \tag{9}$$

Where D (P/R) represents the part torsor.

STEP 4: Compatibility relations

$$0 = -J\,(r_x, P_{32}, P_{12}) + J\,(r_x, P_{22}, P_{11}) + a_{32} - a_{12} + a_{22} - a_{11} \tag{10}$$

$$0 = -J\,(r_x, PL_{100}, PL_{101}) + J\,(r_x, P_{22}, P_{11}) + a_{100} - a_{101} + a_{22} - a_{11} \tag{11}$$

$$0 = -J\,(r_y, P_{32}, P_{12}) + J\,(r_y, P_{22}, P_{11}) + b_{32} - b_{12} + b_{22} - b_{11} \tag{12}$$

$$0 = -J\,(r_y, PL_{100}, PL_{101}) + J\,(r_y, P_{22}, P_{11}) + b_{100} - b_{101} + b_{22} - b_{11} \tag{13}$$

$$0 = -J\,(r_z, P_{32}, P_{12}) + J\,(r_z, P_{22}, P_{11}) + c_{32} - c_{12} + c_{22} - c_{11} \tag{14}$$

$$0 = -J(r_z, PL_{100}, PL_{101}) + J(r_z, P_{22}, P_{11}) + c_{100} - c_{101} + c_{22} - c_{11} \quad (15)$$

$$0 = -J(t_x, P_{32}, P_{12}) + J(t_x, P_{22}, P_{11}) + u_{32} - u_{12} + u_{22} - u_{11} \quad (16)$$

$$0 = -J(t_y, P_{32}, P_{12}) + J(t_y, P_{22}, P_{11}) + v_{32} - v_{12} + v_{22} - v_{11} \quad (17)$$

$$0 = -J(t_z, P_{32}, P_{12}) + J(t_z, P_{22}, P_{11}) + w_{32} - w_{12} + w_{22} - w_{11} \quad (18)$$

From the above system of equations (eq. no 10 to 18) the desired result are obtained by plugging in the data values from table 1 and chain of deviations can be evaluated.. Also by applying the angular conditions and relative positioning conditions, system of equations (eq. no 10 to 18) can be solved. Solving the above system of equation the result comes out to be:

$$J(r_x, P_{32}, P_{12}) = J(r_x, PL_{100}, PL_{101}) \quad (19)$$

$$J(r_y, P_{32}, P_{12}) = J(r_y, PL_{100}, PL_{101}) \quad (20)$$

$$J(r_z, P_{32}, P_{12}) = J(r_z, PL_{100}, PL_{101}) \quad (21)$$

$$J(t_x, P_{32}, P_{12}) = J(r_x, P_{32}, P_{12}) - 0.32 \quad (22)$$

$$J(t_y, P_{32}, P_{12}) = J(r_y, P_{32}, P_{12}) - 0.32 \quad (23)$$

$$J(t_z, P_{32}, P_{12}) = J(r_z, P_{32}, P_{12}) - 0.32 \quad (24)$$

The above equations give the elements of $T_{(P32/P12)}$. Equation no. 19 to 21 gives the angular components of the torsor. They are dependent on the components of gap torsor of planes 100 and 101. Similarly equations 22 to 24 which are derived from equation 16 to 18 by plugging in the values from table 1 gives the translation components of the torsor.

The model of indeterminate requires more computation in comparison to the algorithmic approach. Algorithmic approach is an iterative process which leads to tolerances just like a closed loop system. In this approach the indeterminate values are taken as zero. In the second approach the number of equations to deal with is greater in number. There are loop equations, compatibility equations and a resolving technique which gives the part tolerances. In this method tolerance evaluation between the interacting surfaces is dependent on components of other torsors. On the other hand in the first approach the calculation of torsor for each interacting surface is independent. The benefit of model of indeterminate is that we can determine the indeterminate values in the torsor. There effect can be incorporated in the design process. In the second approach there might be a fact we come across an indeterminate system. Then different conditions and assumptions are applied to solve that system.

5. CONCLUSION

Primary objective of tolerance analysis is to highlight where and how the variations are occurring in the system. By applying the algorithmic approach in the tolerance analysis the absolute values are obtained. These values can act as a feedback system for the process planning or if the changes are to be made in the part design. On the other hand by applying model of indeterminate for the tolerance analysis a system of equations is obtained which can help us to obtain the indeterminate values and chain of deviations. In case of an indeterminate system a lot of assumptions are required that will affect the end result. This is not the case in algorithmic approach. The indeterminate values can be determined in model of indeterminate while they are taken zero in algorithmic approach. Model of indeterminate requires resolution of system of equations while it is not there in algorithmic approach.

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