

ANFIS Based Modeling for Processing Variables' Effects on Coating Properties in Plasma Spraying Process

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ABSTRACT

In order to model the effect of processing variables including primary gas flow rate, stand-off distance, powder flow rate, and arc current on the plasma spraying coating properties including thickness, porosity and micro-hardness, adaptive neural fuzzy inference system (ANFIS) and neural network based models are proposed to understand the spraying process and estimate process parameters. In order to overcome the difficulty of small size of sample data, bootstrap method is applied for the resampling technique and cross validation is applied for the performance evaluation. The ANFIS model and NN model are compared on the performance metrics of 1) mean square error (MSE), and determination coefficient (R^2). The comparisons illustrated that ANFIS based modeling showed significant superiority than the other approach. This may be due to the fact that ANFIS combines the strength of NN's learning capability and fuzzy logic's knowledge interpretation ability. With this ANFIS model and identified control rules, feedback control strategy can be effectively implemented to regulate the coating quality in plasma spraying process.

1. INTRODUCTION

Plasma spraying process is typically for producing thermal barrier coatings (TBCs), which have a very low thermal conductivity, and a high melting point, thus insulating and protecting underlying super-alloys from exposure to the high temperatures. In order to operate in the most demanding high-temperature environment of aircraft and industrial gas-turbine engines, TBCs have complex structure as shown in Figure 1. It comprises of metal and ceramic multilayers, insulate turbine and combustor engine components from the hot gas stream, and improve the durability and energy efficiency of these engines [1].

The process of plasma spraying is shown in Figure 2. The plasma spraying gun generates the heat by an electric arc. A feedstock material is heated and propelled as individual particles or droplets onto a substrate. When being heated, the material phase is transformed to a plastic or molten state; the molten particles are accelerated by a compressed gas stream and strike to the substrate. As the sprayed particles hit upon the substrate, they flatten, cool, and build up layer-by-layer thin splats that conform and adhere to the irregularities of the prepared substrate and to each other.

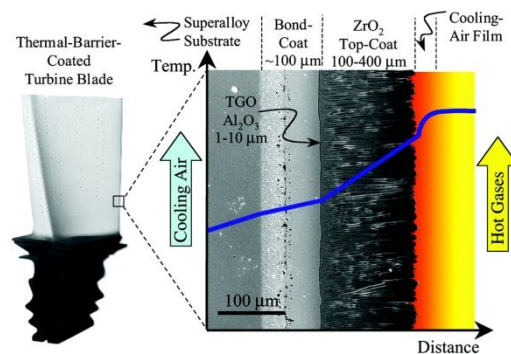


Figure 1. Layers of thermal barrier coating on a turbine blade [1].

In the coating process, cracks, pores, and splats on the TBCs need to be controlled, because it results significantly lower thermal conductivity than the bulk material. The defected microstructure also causes nonlinear elastic modulus. Qualities on coating properties including thickness, hardness, porosity rate etc. are the control objective through tuning the processing variables including plasma gas choice, flow rates of plasma gases, size of nozzle, type of injection, feed rate of powder, feedstock powder particle size distribution, morphology of powder

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etc. However, this process control is a formidable challenge because of two reasons: 1) the complicate interaction between the control variables and their effects on the growing of TBCs, the nature of plasma spraying is not fully understood; and 2) plasma spraying process has the characteristics of short run. Most of the gas-turbine engines are mass-customized, which results to smaller lot sizes, shorter lead times and less available process data to construct a control chart. The repeatability of the coating and a coating process is the problem under investigation, but from the state-of-art on control of plasma spraying, breakthrough has not been reported yet.

Understanding the processing variables' effects on the coating properties is the very first step to effectively control the process. Initiated by these, this article aims at identifying of the plasma spraying process. Because of the difficulty of modeling plasma spraying using classic control theory such as state space approaches, artificial intelligence based model, specifically ANFIS or NN based model, is applied to identify the pattern between the processing variable and desired output properties.

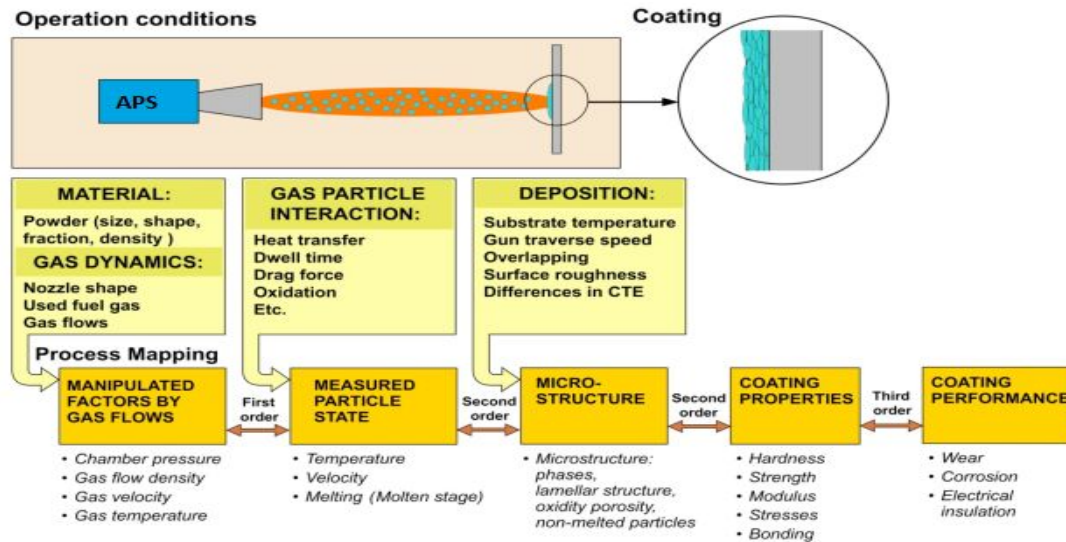


Figure 2. Process of plasma spraying process [2].

The rest of this paper is organized as follows: Section 2 surveys related researches on modeling and control of plasma spraying process, research gap is summarized at the end of this section. Section 3 discusses about the proposed methodology based on ANFIS modeling, bootstrap and k-fold cross validation (KCV). Section 4 describes the case study for validating the proposed approach. Section 5 presents the results and discussions. Section 6 concludes the research and outlines the future direction.

2. LITERATURE REVIEW

On the process model, the modeling can be conducted on 1) the relationship between the processing variables and the in-flight particle states (temperature, velocity and melting states), or 2) the relationship between the processing variables and the final coating properties. The first kind of models are mainly for the purpose of real time control of the thermal spraying, since the particle states can be measured in an in-situ way with the in-flight diagnostics sensors such as DPV 2000[®] [3], Accuraspray[®] [3]etc. However, as shown in Figure 2, there still a gap and uncertainty on the relationship between the particle states and coating deposition structure/properties. On the second kind of models, the coating properties cannot be measured on-line when spraying. Thus these models cannot be directly applied on the process since they involves the measurement delay. Current applications have been conducted on regular regression analysis [4], stepwise regression analysis [5], neural network [6, 7], or fuzzy logic [7].

From the above literature readings, it is identified that when controlling the thermal spraying processes, the current literatures lack enough consideration on the modeling and control: How can we involve uncertainty of the control parameters in the modeling of thermal spraying? Can we generate control rules for modeling the plasma spraying process? These issues are the research questions which will be addressed in the following sections.

3. METHODOLOGIES

3.1. DATA COLLECTION BASED ON DESIGN OF EXPERIMENT

The study starts from data collection with design of experiment (DOE) techniques. DOE permits researchers to study behaviors under conditions in which independent variables vary simultaneously, so the researchers can investigate the joint effect of two or more factors on a dependent variable [13]. The DOE also facilitates the study of interactions, illuminating the effects of different conditions of the experiment on the identifiable subgroups of subjects participating in the experiment. In reference [8], the authors listed five categories of experimental problem according to their objectives: 1) treatment comparisons, 2) variable screenings, 3) response surface exploration, 4) system optimization, and 5) system parameter robustness. This study falls to the categories of 3) and 5). Thus, we want to apply one of the DOE techniques- response surface methodologies (RSM) for experiment design. It can cover a wide range of variables with less number of experiments.

After we collect the data with DOE, the model between the processing variables and the coating properties will be identified with adaptive neuro fuzzy inference system (ANFIS) and neural network (NN) respectively.

3.2. ANFIS BASED MODELING

ANFIS [9] is a class of adaptive networks that functions as a fuzzy-inference system. An ANFIS architecture with Sugeno-type fuzzy inference is shown in Figure 3.

In this structure, the circle nodes represent fixed nodes and the square nodes represent adaptive nodes. The square nodes have parameters which can be updated according to the training data and the gradient based learning procedure.

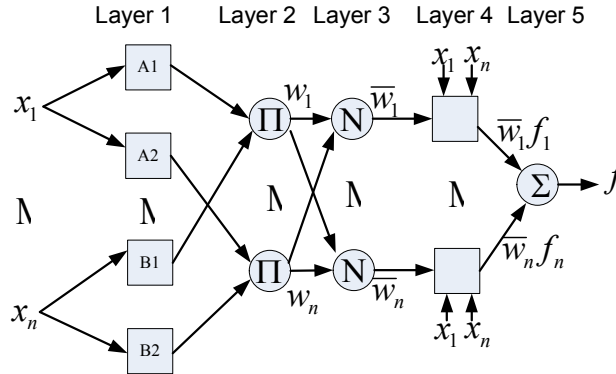


Figure 3. ANFIS architecture [9].

Layer 1: every node in this layer has the node function $O_i^1 = \mu_{A_i}(x)$. Usually $\mu_{A_i}(x)$ is chosen as the bell shape

function such as $\mu_{A_i}(x) = \frac{1}{1 + \left[\left(\frac{x - c_i}{a_i} \right)^2 \right]^{b_i}}$ or $\mu_{A_i}(x) = \exp \left\{ - \left(\frac{x - c_i}{a_i} \right)^2 \right\}$, where $\{a_i, b_i, c_i\}$ is the parameter set.

Parameters in layer are referred to as premise parameters.

Layer 2: every node in this layer is circle node labeled Π which multiplies the incoming signals ($w_i = \mu_{A_i}(x) \times \mu_{B_i}(y)$) and sends out the product.

Layer 3: every node in this layer is circle node labeled N . The i th node calculates the ratio of i th rule's firing strength to sum of all rules' firing strengths: $\bar{w}_i = \frac{w_i}{w_1 + w_n}$.

Layer 4: every node in this layer is a square node with node function $O_i^4 = \bar{w}_i f_i = \bar{w}_i (p_i x + q_i y + r_i)$, $\{p_i, q_i, r_i\}$ is the parameter set, parameter sets are referred as consequence parameters.

Layer 5: the single node in this layer is a circle node labeled Σ which computes the overall output as the summation of all incoming signals, $O_1^5 = \sum_i \bar{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i f_i}$.

In the neuro-fuzzy inference system, it requires two major types of learning: structure learning is used first to find the appropriate structure and fuzzy logic rules of a neuro-fuzzy system; and parameter learning is then used to fine-tune the parameters [10]. Identification of fuzzy rule is the most important aspects in design of fuzzy inference system. Construction of fuzzy logic rules from numerical data consists of two procedures: 1) fuzzy partitioning of the input spaces and/or out spaces; and 2) identification of a fuzzy logic rules for each fuzzy subspaces. Parameters learning methods: gradient descent-based learning algorithms (e.g. error back propagation algorithm), reinforcement learning and approximate least square estimator (LSE) etc.

For the detail of construction of ANFIS and learning algorithms to tune the structure and parameters, the readers can refer to [9, 10].

3.3. NN BASED MODELING

The goal of neural network modeling was to predict the outputs of coating properties using a function of the inputs, which were processing variables that would act upon the output. Desired network architectures were built containing a few hidden layers and hidden nodes for a good prediction of the coating properties. In the neural network modeling process, two aspects of information need to be decided: (1) the neural network's topology including input variables and output variables, number of neurons in each layer, types of neuron functions in each layer; and (2) weights and biases with learning and training functions used in the neural network model [11]. The input was propagated forward through the network to compute the output value. The error is calculated based on the difference between the calculated output value and the desired value. In order to get the mean square error (MSE) between the actual and desired output values as close as possible to zero, back propagation algorithm was applied by adjusting the weights and biases associated with each link of the network [12]. During the backward pass, the error terms were computed when the hidden units and the weights and biases were updated. The output was then compared to the desired output and the MSE was computed. If the error was zero or close to preset values, the network training process stopped. Otherwise desired weights and biases would be searched with different learning and training functions such as gradient descent algorithm etc. [12].

After deciding the NN topology and training the NN weights and biases, the network model is completely determined, and the output OUT_p , as the predicted coating properties, can be expressed by a function $f(X, W, B)$ of the input data $X = [x_1, \Lambda, x_m]^T$, network parameters weights vector $W = [\lambda_1, \Lambda, \lambda_{H_2}; \beta_{11}, \Lambda, \beta_{H_1 H_2}; \gamma_{11}, \Lambda, \gamma_{H_1 m}]$, and biases vector $B = [b_{11}, \Lambda, b_{1 H_1}; b_{21}, \Lambda, b_{2 H_2}; b_{31}]$. The scalars H_1 and H_2 denote the number of nodes in hidden layers 1 and 2 of the network, respectively. $g_1(x)$, $g_2(x)$ and $g_3(x)$, are neuron functions attached to nodes in hidden layer1, layer2 and output layer. The neural network can be interpreted as a parametric nonlinear regression of OUT on X . OUT_p can be calculated as equation

$$OUT_p = f(X, w) = g_3 \left(\sum_{h_2=1}^{H_2} \lambda_{h_2} g_2 \left(\sum_{h_1=1}^{H_1} \beta_{h_2 h_1} g_1 \left(\sum_{i=1}^m \gamma_{h_1 i} x_i + b_{1 h_1} \right) + b_{2 h_2} \right) + b_3 \right).$$

For the detail of construction of NN model and learning algorithms, the readers can refer to [11, 12].

3.4. BOOTSTRAP AND K-FOLD CROSS VALIDATION

The dataset available with DOE techniques usually represents a finite-sized sample set from a population with an unknown probability distribution. The traditional methods for training and testing ANFIS or neural network model call for splitting the dataset into a training subset and a testing subset. The training data subset is used to calculate the weights of the network and the test data subset is used to assess the performance of the model. When the size of the available data is small, the simple training–test data splitting method is not effective. Even with a large size data, the performance of the model cannot be estimated on all future samples presented to the network if a simple training–test data splitting method is used. This is clearly impossible unless the underlying probability distribution that the training samples are drawn from is exactly equal to the probability distribution from which the future samples are drawn. There are a number of schemes for addressing this problem and the most suitable for the size of dataset used in this study, are bootstrap [13] and k-fold cross validation (KCV) [13].

3.4.1. BOOTSTRAP

The bootstrap methods provide a direct computational way of assessing uncertainty, by sampling from the training data [13]. Suppose that we have a model that fits to a set of training data. We denote the training set by

$Z = (z_1, z_2, \Lambda, z_N)$ where $z_i = (x_i, y_i)$. Bootstrap approach will randomly draw datasets with replacement from the training data by B times, each sample the same size as the original training set. This operation will produce B bootstrap datasets, Then the model will be refitted to each of the bootstrap datasets, and examine the behavior of the fits over the B replications. Suppose $s(z_b^*)$ is any quantity computed from the data Z, for example, the prediction at some input point. That estimation can be thought of as a Monte-Carlo estimate of the quantity under sampling from the empirical distribution function \hat{F} for the data (z_1, z_2, Λ, z_N) . Evaluate the bootstrap replication corresponding to each bootstrap sample: $\hat{\theta}^* = s(z_b^*)$, $b=1, \Lambda, B$. Estimate the standard error $se_B(\hat{\theta}^*)$ by the sample standard error of the B replicates $se_B(\hat{\theta}^*) = \left[\frac{1}{B-1} \sum_{b=1}^B (\hat{\theta}^*(b) - \hat{\theta}^*(\bullet))^2 \right]^{1/2}$, here $\hat{\theta}^*(\bullet) = B^{-1} \sum_{b=1}^B \hat{\theta}^*(b)$.

3.4.2. K-FOLD CROSS VALIDATION

KCV randomly divides the available data into k equal size and mutually exclusive partitions (or folds). For a k-fold cross validation k neural networks are trained with a different fold used each time for the validation, while the other k-1 folds are used for the training. The choice of k influences the ratio of data used for training and validating with an optimal value of k in the range 5–10. The performance obtained using a KCV procedure is less biased than the one obtained using a simple training–validating data splitting method. The KCV procedure requires that the number of data samples is a multiple of the number of folds. Five-fold cross validation was used in this study. The available data was randomly partitioned into five mutually exclusive groups. The partition pairs were used to train five prediction models. Figure 4 illustrates the implemented five-cross validation procedure. All the five ANFIS/NN models have the same architecture but differ by the values for their network connection weight vectors.

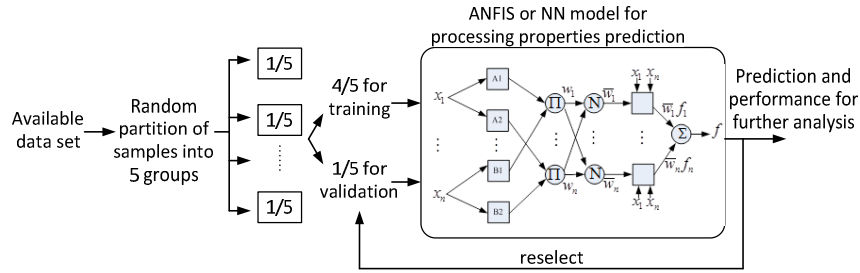


Figure 4. ANFIS training and testing using five-fold cross validation procedure.

The mean squared prediction error is calculated for each model as $MSPE_n = \frac{1}{N_{te}} (d_n(i) - d_{pn}(i))^2$, $n=1, \Lambda, 5$, where d_n is the vector of actual coating property values used during the testing of model n, d_{pn} is the vector of coating property values predicted by network n, and $N_{te} = 5$ is the number of data used for network testing. The cross validation error (CVE) is defined as $CVE = \frac{1}{5} \sum_{n=1}^5 MSPE_n$.

4. CASE STUDY AND RESULTS

In order to illustrate the proposed modeling and validation approach on plasma spraying, a case study was developed on the data was collected by previous studies [4]. The following processing variables of plasma spraying were taken into consideration: primary gas flow rate (G), stand-off distance (D), powder flow rate (P), and arc current (A). The responses considered were thickness (Th), porosity (Pr), and micro-hardness (H) of the coatings. Primary gas flow rate was varied between 7.866×10^{-4} and 11.8×10^{-4} m³/s. The stand-off distances to create the coatings were kept in between 0.150 and 0.200 m. The powder flow rate was varied between 3.775×10^{-3} and 7.7550×10^{-3} kg/s. The range for arc current was considered from 400 to 500 A. Three levels were considered for each of the input parameters. The ranges of the input parameters had been decided based on the manufacturer's recommendation, as those could be machine-dependent. Experiments had been conducted to generate input–output data using a 3 MB Sulzer Metco plasma spray setup. Low-carbon (C-20) steel was taken as the substrate material, and Ni-5 wt% Al alloy powder (450 NS, Sulzer Metco) had been used as the coating material. The plasma spraying

trials had been designed using a central composite design (CCD) based RSM methodology and the collected data is as Table 1.

Table 1. Central composite design of experiments with measured responses [4].

Processing Variables				Coating Properties		
G ($\text{m}^3/\text{s} \times 10^{-4}$)	D (m)	P ($\text{kg}/\text{s} \times 10^{-3}$)	A (ampere)	Th (μm)	Pr (%)	H (Hv_{100})
7.866	0.15	3.775	400	705	11.2	236.1
11.8	0.15	3.775	400	780	11.1	180.9
7.866	0.2	3.775	400	737	6.94	251.9
11.8	0.2	3.775	400	790	8.69	201.9
7.866	0.15	7.755	400	670	8.16	254.5
11.8	0.15	7.755	400	905	9.65	185.1
7.866	0.2	7.755	400	417	8.89	248.8
11.8	0.2	7.755	400	580	10.8	208.4
7.866	0.15	3.775	500	487	7.29	220.9
11.8	0.15	3.775	500	910	8.93	184.0
7.866	0.2	3.775	500	157	9.27	260.3
11.8	0.2	3.775	500	90	10.3	228.5
7.866	0.15	7.755	500	243	7.11	156.8
11.8	0.15	7.755	500	180	8.71	205.4
7.866	0.2	7.755	500	207	9.54	238.9
11.8	0.2	7.755	500	87	8.98	201.0
7.866	0.175	5.662	450	180	8.47	198.0
11.8	0.175	5.662	450	450	11.4	189.8
9.833	0.15	5.662	450	180	10.5	166.2
9.833	0.2	5.662	450	107	9.9	174.3
9.833	0.175	3.775	450	73	10.2	144.8
9.833	0.175	7.755	450	103	7.61	158.5
9.833	0.175	5.662	400	73	11.8	146.5
9.833	0.175	5.662	500	437	11.7	210.4
9.833	0.175	5.662	450	83	10.5	188.2

4.1 RESULTS AND DISCUSSIONS

With the data collected in Table 1, the proposed modeling approaches were conducted. Section 4 will present the results and discussions.

Further goodness-of-fit statistics for the model predictions in arithmetic scale were performed by using statistical parameters such as the determination coefficient R^2 and MSE.

a) Determination coefficient- R^2 : $R^2 = 1 - \left[\frac{\sum_{i=1}^N (\hat{y}_i - \bar{y})^2}{\sum_{i=1}^N (y_i - \bar{y})^2} \right]$. The R^2 is a value between 0 and 1, and

it is a measure of correlation between the predicted and the measured values, thus determining accuracy of the fitting model (the higher R^2 , the higher accuracy).

b) Mean Square Error (MSE): $MSE = \frac{1}{n} \sum_{t=1}^n (y_t - y_{t_{target}})^2$. The y_t is actual value in the t^{th} production run, the

$y_{t_{target}}$ is target value. The smaller the MSE is, the better the modeling result is.

The ANFIS based models were compared with NN models on performances of MSE and R^2 with bootstrap strategy. In these comparisons, the NN models are four-layer forward with the 4-60-60-1(neurons in each layer) structure. The weights and biases as well as the neuron parameters are trained with the back propagation approaches. The summarization of the results is as below Tables 2 and 3. In these two tables, the data was re-sampled for 50 times. In each sampling, 20 sets of data (80 percent of data) were used for modeling, and 5 sets of data (20 percent of data) were used for validation. We can see that ANFIS has better R^2 and MSE performances than NN on the modeling stage.

Table 2. Comparison on the R^2 through bootstrap on thickness, porosity, and micro-hardness.

	Thickness		Porosity		Micro-hardness	
	Model	Validate	Model	Validate	Model	Validate
ANFIS	1	mean:0.57 se: 0.3308	1	mean: 0.42 se: 0.362	1	mean: 0.47 se: 0.351
NN	0.84	mean: 0.512 se: 0.3265	0.867	mean: 0.52 se:0.31	0.887	mean: 0.56 se:0.373

Table 3. Comparison on the MSE through bootstrap on thickness, porosity, and micro-hardness.

	Thickness		Porosity		Micro-hardness	
	Model	Validate	Model	Validate	Model	Validate
ANFIS	0.005	mean: 8.09e4 se: 8.99e4	0.005	mean: 23.67 se: 19.15	1.3e-5	mean:1.02e4 se:1.1e4
NN	1.94e3	mean: 1.82e5 se: 1.03e5	0.32	mean: 89.50 se: 10.24	127.14	mean: 4.12e4 se: 5.99e3

The regression plots for the ANFIS modeling results on (a) thickness, (b) porosity and (c) micro-hardness are illustrated as below Figure 5 (a) to (c). Figure 6 (a) to (c) are the regression plots for NN modeling.

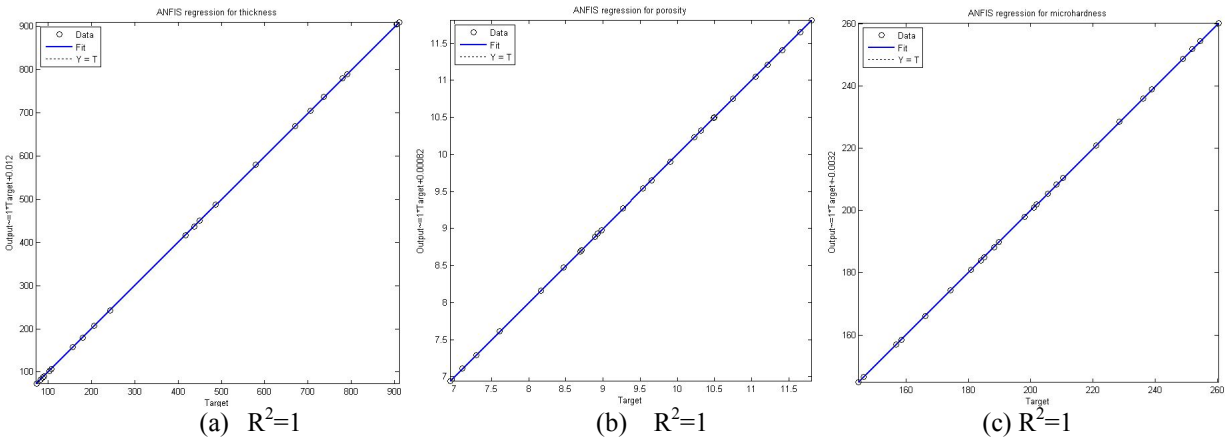


Figure 5. Regression plots for the ANFIS modeling on (a) thickness, (b) porosity and (c) micro-hardness.

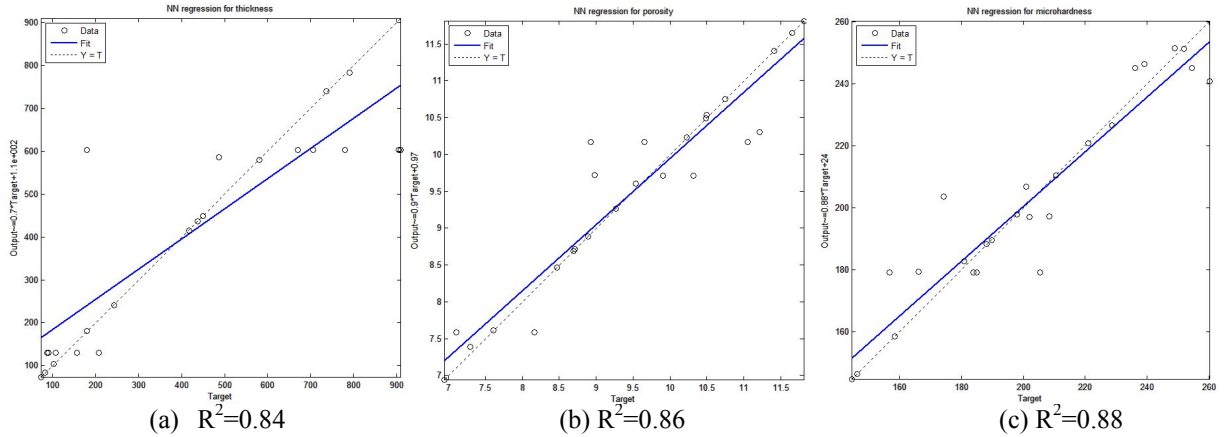


Figure 6. Regression plots for the NN modeling on (a) thickness, (b) porosity and (c) micro-hardness.

The ANFIS based modeling was compared with NN based approach on cross validation performances of MSE. The summarization of the results is as below Table 4.

Table 4. Comparison on the CVE performance on thickness, porosity, and micro-hardness.

	Thickness	Porosity	Micro-hardness
ANFIS	1.32e5	51.99	2.16e4
NN	1.22e5	4.09	1.95e3

From these tables and figures, we see that ANFIS achieved the better results on modeling the effects of processing parameters on the final coating properties. In this stage, the dataset size is relatively larger than the validation stage (80% vs 20%). However, in the validation stage, NN shows better performance. This may be caused due to the uncertainty in the model since there are only 5 pieces of data for validation. Neural network and fuzzy logic are two complimentary techniques. NN can learn from data and feedback, while lacking understands of the knowledge or pattern. Also, it is difficult to develop an insight about the meaning associated with each neuron and each weight. Fuzzy logic is easy to be understood in that it uses the linguistic variables and the structure of “IF-THEN” rules. However, fuzzy logic is short of learning capability. ANFIS combines advantage of these two techniques.

4.2. ANFIS Model Rules between Processing Variables and Coating Properties

As mentioned in Section 3.2, identification of fuzzy rule is the most important aspects in design of fuzzy inference system. ANFIS models also identified rules between processing variables and coating properties. In Figure 7, the surface views about these rules are illustrated. Due to the page limitation, only the model rules between coating properties and processing variables of G and D are presented here.

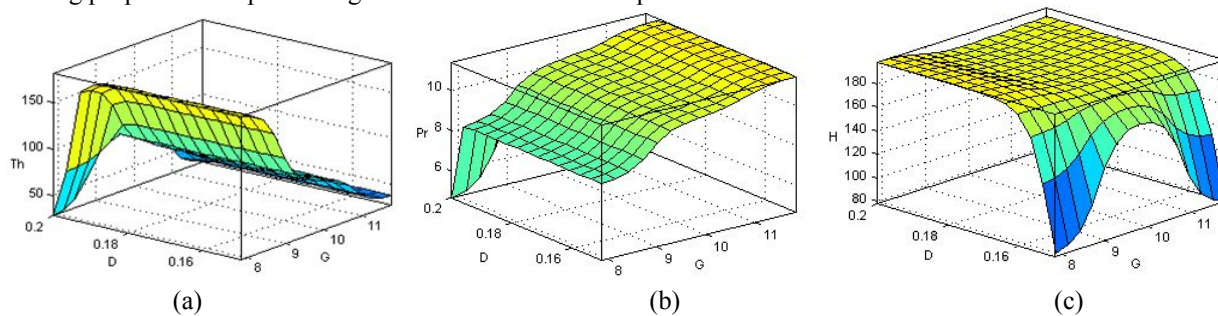


Figure 7. Surface views of the rules of the ANFIS models on (a) thickness, (b) porosity and (c) micro-hardness.

5. CONCLUSIONS AND FUTURE DIRECTIONS

In order to identify the effect of processing variables, including (primary gas flow rate, stand-off distance, powder flow rate, and arc current), on the coating properties including (thickness, porosity and micro-hardness), ANFIS and NN based models were to understand the process and to estimate process parameters. On the modeling performance, we saw that ANFIS indeed improved the performance than 1) multiple variable regression and 2) neural network on MSE and R^2 . This may be due to the fact that ANFIS combines the strength of NN’s learning capability and fuzzy logic’s knowledge interpretation ability.

For future research, further investigations from the following aspects are suggested:

- 1) The models that we developed in this research are all multi-input single-output (MISO) models. Further research will extend the plasma spraying models to multi-input multi-output (MIMO) model.
- 2) Integrate the proposed modeling approaches with effective feedback control algorithms to regulate the plasma spraying process.

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