A Model for Integrating Shipment Consolidation and Pricing Decisions in Perishable Product Supply Chains

Jing Chen[1], Ming Dong[2*], and F. Frank Chen[3]

[1]Sino-US Global Logistics Institute
Shanghai Jiao Tong University
1954 Huashan Road Shanghai,
200030, P.R. China

[2]Antai College of Economics & Management
Shanghai Jiao Tong University
535 Fahua Zhen Road, Shanghai,
200052, P. R. China

[3]Department of Mechanical Engineering
The University of Texas at San Antonio
San Antonio, Texas 78249-0670,
USA

ABSTRACT

This paper focuses on a simple perishable product supply chain with a vendor and multiple retailers. These retailers, densely dispersed in a distribution zone, are sensitive to price, delivery time and product quality. With the aim of optimizing vendor's average expected profit during a shipment consolidation cycle, an analytical model is proposed for this problem. The upper unbound expressions of optimal time policy and fresh-keeping cost are given based on a certain range of time parameter. Then our theoretical findings are verified by using a numerical case. Some useful managerial insights are obtained based on analyzing the sensitivity of this model on major parameters.

1. INTRODUCTION AND RELATED LITERATURE

Because transportation cost increasingly accounts for a larger proportion of logistics cost for perishable goods, shipment consolidation is very important for vendors. The practice is to combine small-size shipments into a larger load based on a policy aiming to benefit from economy of scale associated with transportation costs. Currently, three such consolidation shipment policies are commonly seen in practical applications and discussed in the literatures (Higginson and Bookbinder [1]; Cetinkaya [2]) which are quantity-based, time-based, and time-and-quantity-based policies. In addition, due to the perishable characteristics of products, products often cannot be stored for too long period of time and customers are very sensitive for their price, delivery-time and quality. Thus, in order to obtain economy of transportation scale, the vendor affect the demand by changing product price.

There is significant published literature focused on the shipment consolidation, which can be divided mainly into the pure shipment consolidation work and the integrated shipment consolidation work. The pure shipment consolidation problem only seeks minimization of the inventory cost. Early academic treatments are mostly based on simulation modelling, such as the work of Masters [3], Jackson [4], Cooper [5], Closs and Cook [6], Higginson and Bookbinder [7] and Lieb and Randall [8]. Centikaya and Bookbinder [9] took quantitative approach by applying renewal theory to model the consolidation shipment problem using the quantity-based policy and the time-based policy, respectively. And they gave the explicit expressions of the policy parameters for the cases of private carriage and common carriage under Poisson demands. Then, based on this research, Mutlu et al. [10] and Ulku et al. [11] discussed the shipment consolidation problem from cost and profit perspectives, respectively. Considering the effect of product price on the demand rate, Ulku and Bookbinder [12] modelled and maximized a vendor's expected profit rate when the prices charged depending on the arrival times of orders. The integrated shipment consolidation problem optimizes the total cost of replenishment and inventory. Considering Poisson demand, Centikaya et al. [13-15] formulated this problem into three stochastic models by applying quantity-based policy, time-based policy and quantity-and-time-based policy, respectively. Then, Chen et al. [16] also modeled this problem to compare the operation results based on different shipment consolidation policies. Moreover, a renewal theoretical model with quantity-based policy under the common random order arrivals in random sizes was proposed by Centikaya et al. [17]. More recently, many papers analysed the integrated consolidation shipment problem in the perspective of the profit rate. Hong and Lee [18] considered a single-item inventory system and developed a mathematical model with the time-based consolidation policy to obtain the optimal price, replenishment quantity and dispatch cycle for maximizing the total profit. However, these papers all focused on the common products. There is a gap about the joint shipment

* Corresponding author: Tel.: +86-15821929669; Fax: +86-021-52301 193; E-mail: mdong@sjtu.edu.cn

consolidation and pricing decisions for the perishable products in the literature. Thus, considering a single vendor, distributed the perishable goods to retailers by its own fleet of trucks, and multiple retailers, this paper models this problem based on the price-, product quality-and distribution time-sensitive stochastic demand. Exploring the optimal price and consolidation time for the demand is needed. Moreover, how the profit, the optimal consolidation time and the price are affected by model parameters such as cost and capacity factors will be discussed.

The remainder of this paper is organized as follows. In Section 2, details of the problem setting are described and related modelling assumptions are given. Next, the model is developed and solved in Section 3. In Section 4, a numerical example is offered and sensitivity analyses on the optimal solutions are conducted. Finally, concluding remarks and future research are provided in Section 5.

2. PROBLEM DEFINITION AND MODELING ASSUMPTIONS

We consider a pure perishable product's consolidation shipment and pricing problem encountered by a vendor, which is shown according to a real food company in China. The pictorial explanation of the problem is depicted in Figure 1. The vendor announces the price, quality and delivery time of perishable products which are sold in standard size. The retailers send their demands to the Regional Distribution Center (RDC) which is owned by the vendor if retailers accept the price, quality and delivery time of the products. Then, the vendor consolidates loads according to the real-time order information from RDC and dispatches products to RDC based on the predetermined optimal shipment consolidation cycle. Then, the local delivery to these retailers is implemented by RDC in a predetermined delivery zone. This cycle repeats on a rolling horizon. Moreover, we denote the adjacent departure time of the vendor to be a shipment consolidation cycle (SCL-cycle).

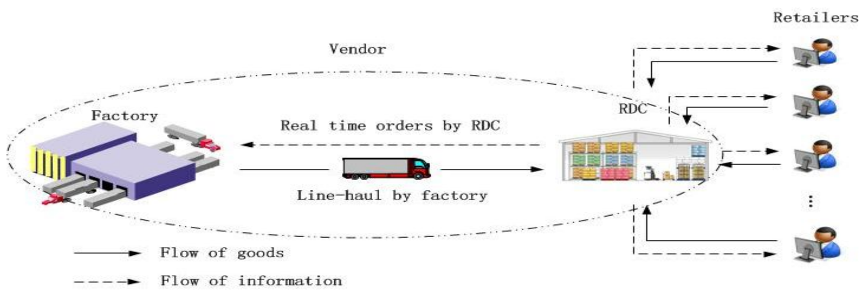

Figure 1. Flow of products and information in the model.

In order to facilitate the modeling, the vendor's transport capacity is assumed to be unlimited. And comparing to the distance of local delivery, the distance between the factory and the RDC is significantly much longer so that the local delivery in this model can be neglected. Besides, the quality of perishable products deteriorates as time elapses. But the vendor can utilize some preservation methods to slow down product's deteriorating rate at the expense of high fresh-keeping cost. To ensure the perishable product's quality is acceptable for the retailers, these products must be dispatched before they reach the minimum accepted quality level. The vendor applies the time-based shipment consolidation policy, so that a particular quoted delivery-time can be guaranteed. The parameters defined in the model are shown in Table 1.

Table 1. The definition of parameters in the model.

Parameters	Descriptions
D_{max}	Maximum potential demand
m_t	Perishable products' quality level at time t
M_{min}	The minimum perishable product's quality level acceptable by retailers
δ_p	Demand sensitivity to price (demand decreases while price increases)
δ_d	Demand sensitivity to delivery-time guarantee (demand decreases while guaranteed delivery time increases)
δ_m	Demand sensitivity to products' quality (demand decreases while perishable products' quality decreases)
V_{max}	Customer's maximum valuation for the perishable product
θ_d	Delivery-time sensitivity of the price (price decreases while guaranteed delivery time increases)
θ_m	Perishable products' quality sensitivity of the price (price decreases while perishable products' quality decreases)
K_w	Fixed order-processing cost for a SCL-cycle (independent of batch size)
K_d	Fixed dispatching cost per vehicle
C_d	Delivery cost per product per unit time
C_w	Handing cost per unit including sorting, loading, unloading, etc. per unit product
C_h	Holding cost per unit product per unit time
μ	Vehicle capacity (number of products)
L	Line-haul time

ρ	Perishable products' quality deteriorating rate
β	Perishable products' quality sensitivity of fresh-keeping cost
T	Length of SCL-cycle
T_t	Delivery-time guaranteed to a customer whose order arrives at vendor at time t
p_t	Price per unit charged to an order arriving at time t
C_f	Fresh-keeping cost per unit product per unit time

The sequence and timing of events in our model are shown in Figure 2. We discuss a price-, delivery time-and products' quality-sensitive demand market, which is similar as the demand market structure in the paper written by UKlu and Bookbinder (2012). The vendor can optimize its expected profit per unit time. At the beginning, it decides the cost C_f (per unit product per unit time) for keeping the perishable products fresh and gives the initial price (p_0) and the initial guaranteed delivery time (T_u), which is the sum of the time of shipment consolidation cycle (T) and line-haul time (L). As the time elapses, the vendor updates the price (p_t) and the guaranteed delivery time (T_t), so that $T_t = T + L - t$, at each unit time t ($0 \le t \le T$). As a consequence, demand accumulates at vendor's site according to a non-stationary Poisson process. Then, at timeT, vendor dispatches the consolidated load to the RDC.

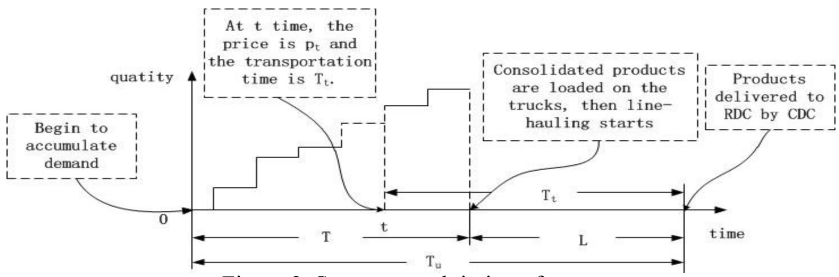

Figure 2. Sequence and timing of events.

We need to determine the optimal fresh-keeping cost, C_f^*, the optimal shipment consolidation cycle length, T^*, and the optimal price, p_t^*, in order to enable the vendor obtain maximum expected profit.

3. PROBLEM FORMULATION AND RESULTS OPTIMIZATION

According to the practice, this paper models the demand arrival rate λ_t at time t as a decreasing function of both price and the delivery time and increasing function of the perishable products' quality, which is seen in Equation (1). We define the products price as a decreasing function about the delivery time and increasing function about the perishable products' quality, which is shown in Equation (2). And the quality function of the perishable products is expressed by the Equation (3), which is a decreasing function about time and increasing function about the fresh-keeping cost. Explicitly, the demand arrival process and the price and perishable products' quality at time t are modeled as:

$$\lambda_t = D_{max} - \delta_p p_t - \delta_d T_t + \delta_m m_t, \delta_p > 0, \delta_d > 0; \delta_m > 0; 0 < t < T \quad (1)$$

$$p_t = V_{max} - \theta_d T_t + \theta_m m_t, \theta_d > 0; \theta_m > 0 \quad (2)$$

$$m_t = m_0 e^{-\frac{\rho t}{\beta C_f + 1}}, \rho > 0; \beta > 0 \quad (3)$$

In Equation (1), D_{max} is the maximum potential demand in market. δ_p, δ_d and δ_m are the parameters that measure the sensitivity of demand to price, guaranteed delivery time and quality, respectively. In Equation (2), V_{max} shows the retailers' maximum willingness to pay for the products and the transportation service. θ_d and θ_m are the parameters that measure the sensitivity of price to guaranteed delivery time and quality, respectively. In Equation (3), the best quality of perishable products is m_0, which is at the beginning of the shipment consolidation cycle. With the passage of time, the quality of products is decreasing as an exponential function proposed by Cai et `k−(2010), where ρ and β are the parameters that measure the sensitivity of quality to time and fresh-keeping cost, respectively.

Combing Equations (1) and (2), we can simplify the demand rate as Equation (4) by letting $A = D_{max} - \delta_p V_{max}$, $B = \delta_d - \delta_p \theta_d$ and $C = \delta_m - \delta_p \theta_m$.

$$\lambda_t = A - BT_t + Cm_t, 0 \le t \le T \text{ and } 0 < L \le T_t \le T_u \quad (4)$$

Observing the Equation (4), the signs of B and C determine the type of market. If B is positive, the retailers are more time-sensitive than price-sensitive, indicating that the retailers are willing to pay additional price for the faster transportation service. However, if B is negative, the retailers are more price-sensitive than time-sensitive, indicating that the retailers are willing to endure a longer transportation time in order to obtain the lower price. In addition, if C is positive, the retailers are more quality-sensitive than price-sensitive, indicating that retailers are willing to pay higher prices for better quality of products. However, if C is negative, the retailers are more price-sensitive than quality-sensitive, indicating that the retailers are willing to pay lower prices at the expense of quality of products.

When the vendor applies the shipment consolidation strategy, it must pay for the inventory cost and transportation cost in order to get the benefit of shipment consolidation during a cycle. And according to the problem definition in Section 2, a new shipment consolidation cycle begins once the consolidated load is dispatched, which is shown as a renewal process. Thus, the average expected total profit per shipment consolidation cycle $E[\Pi(T, C_f)]$ is:

$$E[\Pi(T, C_f)] = \frac{E[\textit{ the expected total revenue}] - E[\textit{the expected total cost}]}{T} \tag{5}$$

And for computing $\Pi(T, C_f)$, we need get the total demand $Q(T, C_f)$ during a shipment consolidation cycle. $Q(T, C_f)$ is a random variable and has a Poisson distribution with mean value λ_t. Thus, we can express the expected total demand consolidated by time T as:

$$Q(T, C_f) = \int_0^T \lambda(p_t, T_t, m_t)\, dt = \int_0^T A - BT_t + Cm_t\, dt = AT - B\left(LT + \frac{T^2}{2}\right) + C\alpha m_0\left(1 - e^{-\frac{T}{\alpha}}\right) \tag{6}$$

During a shipment consolidation cycle, the revenue is $\lambda_t p_t$ at time t. And we denote $\alpha = (\beta C\_f + 1)/\rho$. Thus, the expected total revenue ($E[TR]$) is computed in Equation (7).

$$E[TR] = \frac{B\theta_d}{3}T^3 - \frac{BV_{max} + (A - 2BL)\theta_d}{2}T^2 + [V_{max}(A - BL) + (BL^2 - AL - C\alpha m_0)\theta_d - \alpha m_0 B\theta_m]T$$
$$+ \alpha m_0[(A\theta_m + CV_{max}) - (L - \alpha)(C\theta_d + B\theta_m)]\left(1 - e^{-\frac{T}{\alpha}}\right) + C\theta_m \frac{m_0^2\alpha}{2}\left(1 - e^{-\frac{2T}{\alpha}}\right) \tag{7}$$

The total cost includes the inventory cost and the transportation cost. The inventory cost is the sum of the fixed and the variable order-processing cost for an SCL-cycle, the holding cost and the fresh-keeping cost (E[HC]).

$$E[HC] = K_w + C\alpha m_0[C_w - \alpha(C_h + C_f)]\left(1 - e^{\frac{T}{\alpha}}\right) - \frac{B(C_h + C_f)}{3}T^3 + \frac{(C_h + C_f)(A - BL) - BC_w}{2}T^2 + [C_w(A - BL) + C\alpha m_0(C_h + C_f)]T \tag{8}$$

The transportation cost (E[TC]) consists of the fixed vehicle cost and the variable transportation cost.

$$E[TC] = K_d\left\lceil\frac{-\frac{B}{2}T^2 + (A - BL)T + C\alpha m_0(1 - e^{-\frac{T}{\alpha}})}{\mu}\right\rceil - \frac{BC_dL}{2}T^2 + (A - BL)C_dLT + C\alpha m_0C_dL(1 - e^{-\frac{T}{\alpha}}) \tag{9}$$

where $\lceil x \rceil$ is ceiling function which registers the smallest integer greater than or equal to x. To sum up, the average total profit per shipment consolidation cycle is obtained in Equation (10).

$$E[\Pi(T, C_f)] = \frac{B}{3}\varphi_1 T^2 - \frac{1}{2}[(A - BL)\varphi_1 + B\varphi_2]T + \varphi_2(A - BL) - C\alpha m_0\Phi_1 - \alpha m_0 B\theta_m$$
$$+ \frac{\alpha m_0}{T}[C\alpha\varphi_1 + C\varphi_2 + \theta_m(A - BL + B\alpha)]\left(1 - e^{-\frac{T}{\alpha}}\right)$$
$$+ C\theta_m\frac{m_0^2\alpha}{2T}\left(1 - e^{-\frac{2T}{\alpha}}\right) - \frac{K_w}{T} - \frac{K_d}{T}\left\lceil\frac{-\frac{B}{2}T^2 + (A - BL)T + C\alpha m_0(1 - e^{-\frac{T}{\alpha}})}{\mu}\right\rceil \tag{10}$$

where $\varphi_1 = \theta_d + C_h + C_f$, and $\varphi_2 = V_{max} - L\theta_d - LC_d - C_w$

To ensure $E[\Pi(T, C_f)]$ is available, there are three constraint conditions about T. 1) The price must be positive ($p_t \geq 0$). And we denote U_1 to be the upper bound. Because $\frac{\partial^2 p_t}{\partial^2 t} = \frac{\theta_m m_0 e^{-\frac{t}{\alpha}}}{\alpha^2} \geq 0$, we have: a) if $\theta_d\alpha \geq \theta_m m_0$, p_t is a monotone increasing function in $t \in [0, \infty)$. The minimum value of p_t is the point $t = 0$. Thus, $T \leq \frac{V_{max} - C_w + \theta_m m_0}{\theta_d} - L = U_1$. b) if $\theta_d\alpha < \theta_m m_0$, there is the minimum value of p_t is the point $t^* = \alpha\log\frac{\theta_m m_0}{\alpha\theta_d}$. Thus, $T \leq \frac{V_{max} - C_w}{\theta_d} + \alpha\left(1 + \ln\left(\frac{\theta_m m_0}{\theta_d\alpha}\right)\right) - L = U_1$. 2) The demand rate must be positive ($\lambda_t \geq 0$). We denote U_2 to be the upper bound. Because $\frac{\partial^2\lambda}{\partial^2 t} = \frac{Ce^{-\frac{t}{\alpha}}m_0}{\alpha^2} \geq 0$ too, as the analyzing process in 1), we have: a) If $B\alpha \geq Cm_0$, $U_2 = \frac{A + Cm_0}{B} - L$. b) If $B\alpha < Cm_0$, $U_2 = \frac{A}{B} + \alpha\left(1 + \ln\left(\frac{Cm_0}{B\alpha}\right)\right) - L$. 3) The perishable products' quality cannot be lower than M_{min} ($m_T \geq M_{min}$), which is requested by retailers. We denote U_3 as the upper bound in this constraint condition. Thus, we have $U_3 = \alpha\ln\frac{m_0}{M_{min}} - L$. To sum up, we define U, the upper bound on the consolidation cycle length, as $U = \min\{U_1, U_2, U_3\}$.

Our analysis is simplified by replacing $E[\Pi(T, C_f)]$ with what we call a modified function $E_u[\Pi(T, C_f)]$, which is obtained by removing those ceiling functions in Equation (10). Thus, $E_u[\Pi(T, C_f)]$, a continuous function in T and C_f, forms an upper bound function for $E[\Pi(T, C_f)]$.

$$E_u[\Pi(T, C_f)] = \frac{B\varphi_1}{3}T^2 - \frac{1}{2}[(A - BL)\varphi_1 + B\Phi_2]T$$

$$+\frac{\alpha m_0}{T}[C\alpha\varphi_1 + C\Phi_2 + \theta_m(A - BL + B\alpha)]\left(1 - e^{-\frac{T}{\alpha}}\right) + C\theta_m\frac{m_0^2\alpha}{2T}\left(1 - e^{-\frac{2T}{\alpha}}\right) - \frac{K_w}{T}$$
$$+ (A - BL)\Phi_2 - C\alpha m_0\varphi_1 - \alpha m_0 B\theta_m \quad (11)$$

where $\Phi_2 = \varphi_2 - \frac{K_d}{\mu}$

It is obvious that the Equation (11) is not convex or concave about T or C_f. Thus, we discuss a special situation, which is $T \le \alpha$. According to the Taylor polynomial of $e^{-\frac{T}{\alpha}}$, we get the upper bound of $E[\Pi(T, C_f)]$ to be $E_u[\Pi(T, C_f)]_{T\le\alpha}$.

$$E_u[\Pi(T, C_f)]_{T\le\alpha} = \frac{B\varphi_1}{3}T^2 - \frac{1}{2}[(A - BL)\varphi_1 + B\Phi_2]T - \frac{K_w}{T} + (A - BL + Cm_0)(m_0\theta_m + \Phi_2) \quad (12)$$

We firstly focus on the modified function $E_u^1[\Pi(T, C_f)]_{T\le\alpha}$. Based on the results, we can extend the findings to the $E[\Pi(T, C_f)]$. Observing the Equation (12), we give the following Lemma.

Lemma1: If $B \le 0$, $E_u[\Pi(T, C_f)]_{T\le\alpha}$ is a decreasing function about C_f. However, If $B > 0$ and $\frac{3}{2B}(A - BL) < \alpha$, $E_u[\Pi(T, C_f)]_{T\le\alpha}$ is a decreasing function about C_f in the range $T \in (0, \frac{3}{2B}(A - BL)]$, but is a increasing function about C_f in the range $T \in (\frac{3}{2B}(A - BL), \alpha]$. Moreover, if $\frac{3}{2B}(A - BL) \ge \alpha$ ($C_f \le \frac{3\rho(A-BL)-2B}{2\beta B}$), $E_u[\Pi(T, C_f)]_{T\le\alpha}$ is a decreasing function about C_f.

Poof: Simply by checking the first-order derivative about C_f, we can get the results above.

According to Lemma 1, we are able to know that the value of fresh-keeping cost (C_f) depends on the relationship of the market (B and $\frac{3}{2B}(A - BL)$) and the character of perishable products (α). When the demand is increasing as the delivery-time ($B \le 0$). To obtain the maximum average expected profit, the vendor doesn't need apply any preservation measures ($C_f = 0$). However, when the demand is decreasing as the delivery-time and the relationship of the market and the character of perishable products is $\frac{3}{2B}(A - BL) < \alpha$, we can get two situations: a) if the time parameter T is less than or equal to $\frac{3}{2B}(A - BL)$, the vendor should not apply any preservation measures ($C_f = 0$). b) if the time parameter T is larger than $\frac{3}{2B}(A - BL)$, the vendor need to pay the fresh-keeping cost to get the maximum average expected profit. Moreover, when the relationship of the market and the character of perishable products is $\frac{3}{2B}(A - BL) \ge \alpha$, the vendor still does not need to apply any preservation measures ($C_f = 0$). Thus, when the vendor does not apply any preservation measures ($C_f = 0$), we have the Equation (13). And by discussing it, we can get the Lemma 2.

$$E_u[\Pi(T, C_f)]_{T\le\frac{1}{\rho}} = \frac{B(\theta_d + C_h)}{3}T^2 - \frac{1}{2}[(A - BL)(\theta_d + C_h) + B\Phi_2]T - \frac{K_w}{T} + (A - BL + Cm_0)(m_0\theta_m + \Phi_2) \quad (13)$$

Lemma2: If $B \le 0$, $E_u[\Pi(T, C_f)]_{T\le\frac{1}{\rho}}$ is concave about T everywhere. Moreover, If $B > 0$, then $E_u[\Pi(T, C_f)]_{T\le\frac{1}{\rho}}$ is concave about T in the range $[0, \min(\sqrt[3]{\frac{3K_w}{B\varphi_1}}, \frac{1}{\rho})]$ and is convex about T otherwise where $T \in (0, \frac{1}{\rho}]$.

Poof: It is easy to obtain the Lemma by analyzing the second-order conditions about T under using the fact that θ_d, C_h, C_f, T, K_w are all greater than or equal to zero and $T \le \frac{1}{\rho}$.

Hence, the optimal solution of $E_u[\Pi(T, C_f)]_{T\le\frac{1}{\rho}}$ is an interior (when it is a concave function) or a boundary solution (when it is a convex function). That is:

$$T_0 = \arg\max\begin{cases} E_u[\Pi(T, C_f)]_{T\le\frac{1}{\rho}}, & B \le 0, 0 < T \le \frac{1}{\rho} \\ E_u(T = 0) \text{ or } E_u\left[T = \min\left(\sqrt[3]{\frac{3K_w}{B\varphi_1}}, \frac{1}{\rho}\right)\right], & B > 0 \\ E_u[\Pi(T, C_f)]_{T\le\frac{1}{\rho}}, & B > 0, otherwise \end{cases} \quad (14)$$

Then, the upper bound of the vehicle' number is $n_v = [\frac{Q(T_0,0)}{C}]$. According to it, we can find the other two solutions T_1 and T_2.

$$T_1 = \arg\max\{E[\Pi(T, 0)], T \in S_1 \cap [0, \tfrac{1}{\rho}]\}, \text{ where } S_1 = \arg\{Q(T, 0) = (n_v - 1)C, 0 \le T \le \min\left(\sqrt[3]{\frac{3K_w}{B\varphi_1}}, \frac{1}{\rho}\right)\} \quad (15)$$

$$T_2 = \arg\max\{E[\Pi(T, 0)], T \in S_2 \cap [0, \tfrac{1}{\rho}]\}, \text{ where } S_2 = \arg\{Q(T, 0) = n_v C, 0 \le T \le \min\left(\sqrt[3]{\frac{3K_w}{B\varphi_1}}, \frac{1}{\rho}\right)\} \quad (16)$$

Proposition 1: When $T \le \frac{1}{\rho}$ and $C_f = 0$, the optimal time parameter T^* is one of T_0, T_1 and T_2, which are obtained by the Equations (14) to (16).

Proof: Because T_0 is the optimal solution of $E_u[\Pi(T,0)]_{T\le\frac{1}{\rho}}$, which is the upper bound for $E[\Pi(T,0)]_{T\le\frac{1}{\rho}}$, we have $E_u[\Pi(T_0,0)]_{T\le\frac{1}{\rho}} \ge E[\Pi(T_0,0)]_{T\le\frac{1}{\rho}}$. And according to Equation (6), $Q'(T,C_f) = A - B(L+T) + Cm_0e^{-\frac{T}{\alpha}} = \lambda_0 \ge 0$, which shows that $Q(T,C_f)$ is increasing in T. Thus, $T_1 \le T_0 \le T_2$. Observing Lemma2, when $E_u[\Pi(T,C_f)]_{T\le\frac{1}{\rho}}$ is concave over $[T_1,T_2]$, there are $E[\Pi(T_0,0)]_{T\le\frac{1}{\rho}} \ge E[\Pi(T_1,0)]_{T\le\frac{1}{\rho}}$ and $E[\Pi(T_0,0)]_{T\le\frac{1}{\rho}} \ge E[\Pi(T_2,0)]_{T\le\frac{1}{\rho}}$. And when $E_u[\Pi(T,0)]_{T\le\frac{1}{\rho}}$ is convex, the optimal solution of $E[\Pi(T,0)]_{T\le\frac{1}{\rho}}$ is a boundary value.

However, if $B > 0, \frac{3}{2B}(A - BL) < \alpha$ and $T \in [\frac{3}{2B}(A - BL), \alpha]$, the vendor should pay for much fresh-keeping cost in order to obtain the maximum average expected profit. Thus, we have the Proposition 2 next.

Proposition 2: If $B > 0$and $T \le \alpha$, the optimal solution of $E_u[\Pi(T,0)]_{T\le\frac{1}{\rho}}$ is $T^* = \frac{3}{2B}(A - BL)$ and $C_f^* = \frac{9B\phi_2(A-BL)^2-8B^2K_w}{9(A-BL)^3} - \theta_d - C_h$ based on the condition $C_f^* \ge \frac{3\rho(A-BL)-2B}{2\beta B}$.

Proof: According to the condition $\frac{3}{2B}(A - BL) \le \alpha$, we can obtain the range of C_f to be $C_f \ge \frac{3\rho(A-BL)-2B}{2\beta B}$. And we obtain the optimal solutions of T and C_f by letting the partial derivatives of $E_u[\Pi(T,C_f)]_{T\le\alpha}$ about T and C_f be zero. Then, we obtain $T^* = \frac{3}{2B}(A - BL)$ and $C_f^* = \frac{9B\phi_2(A-BL)^2-8B^2K_w}{9(A-BL)^3} - \theta_d - C_h$.

Once the optimal solutions of T^* and C_f^* are obtained, it is easily to find the corresponding optimal price (p_t) at each time and the maximum average excepted profit by using Equation (2) and Equation (10), respectively.

4. Numerical analyses

In this section, through some numerical examples, we obtain some insights about the shipment consolidation problem in the perishable products industry. Firstly, we compute a numerical study based on the parameters in Table 2.

Table 2. The parameters values in our model.

D_{max} (units)	V_{max} (\$)	m_0 (units)	M_{min} (units)	δ_p (%)	δ_d (%)	δ_m (%)	θ_d (%)	θ_m (%)	K_w (\$)	K_d (\$)	C_d (\$)	C_w (\$)	C_h (\$)	M (units)	L (days)	P (%)	B (%)
150	150	100	70	0.9	0.5	0.4	0.4	0.3	30	80	15	3	2	100	4	0.1	0.05

By solving the Equation (10) based on these parameters above, we obtain $T^* = 0.41$ days and $C_f^* = \$4.73$, which is smaller than the results in Proposition 2. Thus, the vendor can get the average expected profit to be $1841.60. Moreover, using these dates in Table 2 as foundation, we give the sensitivity analysis about major parameters in our model next. We firstly study the sensitivity of the optimal average expected profit, time shipment consolidation parameter and fresh-keeping cost to the Line-haul time. Given Figure 3, when the Line-haul time increases from 1 day to 8 days, the optimal average expected profit decreases at a faster rate. However, the optimal fresh-keeping cost, which is applied by the vendor, is changing inversely with the Line-haul time. Moreover, it shows that the time parameter (T) shows an erratic behavior. By analyzing the effect of the Line-haul time on our model, the shorter Lin-haul time can make the vendor obtain the better operational benefit.

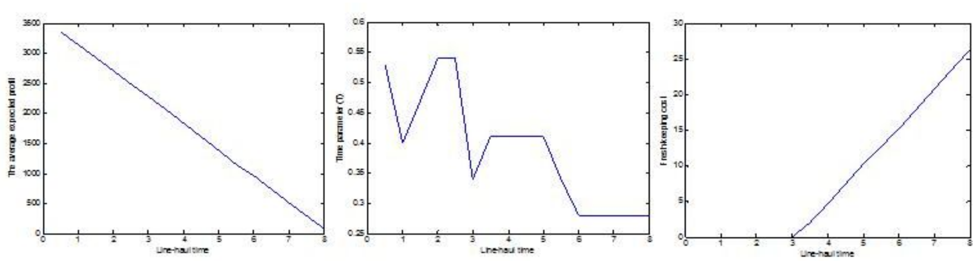

Figure 3. The sensitivity of shipment consolidation problem to the Line-haul time.

Then, we discuss the effect of the market (δ_p, δ_t and δ_m) on the shipment consolidation problem in Figure 4. It is obtained that there is slight effect on the average excepted profit, optimal shipment consolidation time and fresh-keeping cost when demand sensitivity to delivery-time guarantee changes from 0 to 1. But when the parameter of demand sensitivity rate (δ_p) changes from 0 to 1, the average excepted profit decreases faster. And the two optimal decision variables (T and C_f) show a fluctuant increasing process. Thus, the vendor can get bigger average excepted profit on the market, which is not sensitive to price. On the contrary, the average excepted profit keeps increasing when δ_m changes from 0.2 to 1. When the market has little sensitivity to the quality of perishable products (δ_m = 0.2), the two optimal decision variables both reach the maximum value. With the increase of δ_m, the optimal values of two decision variables fall firstly, then go up. It also says that the vendor can obtain a higher average excepted profit on the market, which is more sensitive to the products quality.

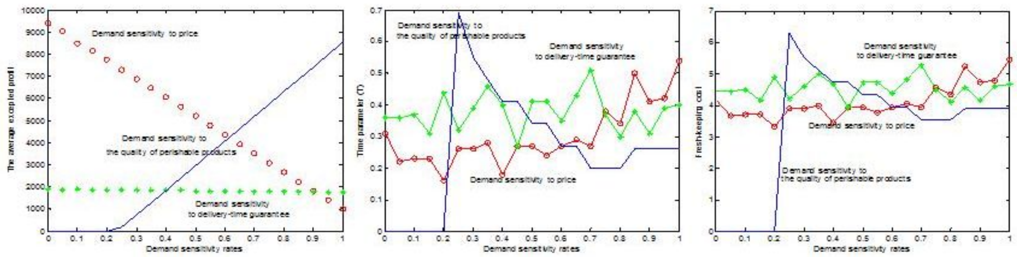

Figure 4. The sensitivity of shipment consolidation problem on demand sensitivity rates.

The price sensitivity rates, price sensitivity to the quality of perishable products (θ_m), and price sensitivity to delivery-time guarantee (θ_d), which is decided by vendor, can affect the results of our model. From Figure 5, when the two rates are both sensitive to price, the vendor can obtain a higher average excepted profit. And in this situation, the benefit of shipment consolidation and the fresh-keeping cost get into full play.

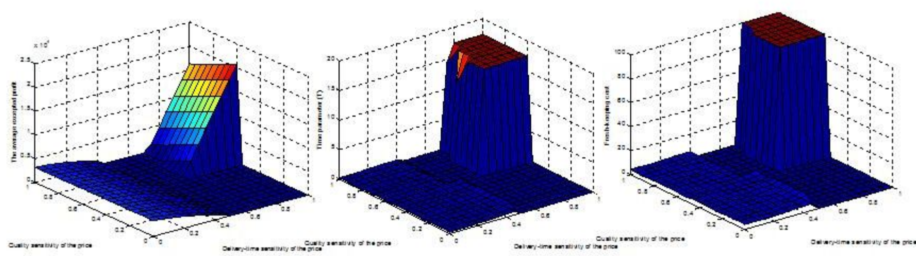

Figure 5. The sensitivity of shipment consolidation problem on price sensitivity rates.

Finally, we explore the sensitivity of our model to products deteriorating rate (ρ) and the fresh-keeping rate (β). For the product with lower perishable speed, the vendor can obtain bigger shipment consolidation time, which generates higher average excepted profit. When the perishable rate of products changing from 0.02 to 0.06, the vendor does not need to input any fresh-keeping cost. Given Figure 6, the higher fresh-keeping rate of perishable product, the higher profit and longer shipment consolidation time the vendor will obtain.

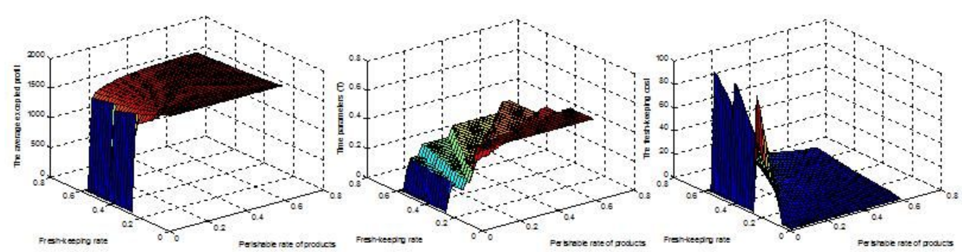

Figure 6. The sensitivity of shipment consolidation problem on products' perishable rate and fresh-keeping rate.

4. CONCLUSIONS

In a simple supply chain situation of the perishable products industry, where a vendor and some retailers, who are sensitive to price, delivery-time guarantee and products quality, we model the shipment consolidation problem for optimizing average excepted profit of the vendor. We found the expressions of optimal time policy and the fresh-keeping cost, which is applied by the vendor, in the condition $T \leq \alpha$. Then, for exploring the effect of model parameters on this problem, we study the sensitivity of shipment consolidation problem on the Line-haul time, demand sensitivity rates, price sensitivity rates, products' perishable rate, fresh-keeping rate, fixed dispatching cost and vehicle capacity, respectively. According to these computed results, we analyze the effect of these parameters in our model on the optimal results.

A useful extension of our current work would be to study the difference of quantity-based policy, time-based policy and quantity-and-time-based policy on the profit for perishable products' shipment consolidation. And the replenishment decision of vendor will be consider into our perishable production shipment consolidation model. Moreover, we will consider the situation that these sensitivity parameters (δ_p, δ_d, δ_m) are different for each retailer. The parameters and some inputting function structures will be studied by the practical data. We will explore the perishable product shipment consolidation problem based on data-driven method in the future.

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REFERENCES

[1] Higginson, J.K. & Bookbinder, J.H., 1995. Markovian decision processes in shipment consolidation. Transportation Science, 29(3), 242–255.

[2] Cetinkaya, S., 2004. Coordination of inventory and shipment consolidation decisions: a review of premises, models, and justification. Applications of Supply Chain Management and E-Commerce Research in Industry, New York, NY, Chapter 1.

[3] Masters, J.M., 1980. The effects of freight consolidation on customer service. Journal of Business Logistics, 2(1), 55–74.

[4] Jackson, G.C., 1981. Evaluating order consolidation strategies using simulation. Journal of Business Logistics, 2(2), 110–138.

[5] Cai, X.Q., Chen, J., Xiao Y.B. and Xu, X.L., 2010. Optimization and coordination of fresh product supply chains with freshness-keeping effort. Production and Operations Management, 19(3), 261–278.

[6] Cooper, M.C., 1984. Cost and delivery time implications of freight consolidation and warehouse strategies. International Journal of Physical Distribution and Materials Management, 14(6), 47–67.

[7] Closs, D.J., Cook, R.L., 1987. Multi-stage transportation consolidation analysis using dynamic simulation. International Journal of Physical Distribution and Materials Management, 17(3), 28–45.

[8] Higginson, J.K. & Bookbinder, J.H., 1994. Policy recommendations for a shipment consolidation program. Journal of Business Logistics, 15(1), 87–112.

[9] Lieb, R. and Randall, H., 1996. A comparison of the use of third-party logistics services by large American manufacturers. Journal of Business Logistics, 17(5), 305–320.

[10] Centinkaya, S., Bookbinder, J.H., 2003. Stochastic models for the dispatch of consolidated shipments. Transportation Research Part B, 37(8), 747–748.

[11] Mutlu, F., Cetinkaya, S., & Bookbinder, J.H., 2010. An analytical model for computing the optimal time-and-quantity-based policy for consolidated shipments. IIE Transactions, 42, 367–377.

[12] Ulku, M.A., Bookbinder, J.H., 2012. Modelling shipment consolidation and pricing decisions for a manufacturer-distributor. International Journal of Revenue Management, 6(1), 62–76.

[13] Ulku, M.A., Bookbinder, J.H., 2012. Optimal quoting of delivery time by a third part logistics provider: The impact of shipment consolidation and temporal pricing schemes. European Journal of Operational Research, 221(1), 110–117.

[14] Cetinkaya, S., Lee, C.-Y., 2000. Stock Replenishment and Shipment Scheduling for Vendor-Managed Inventory Systems. Management Science, 46(2), 217–232.

[15] Cetinkaya, S., Lee, C.-Y., 2002, Optimal outbound dispatch policies: modeling inventory and cargo capacity. Naval Research Logistics, 49, 521–556.

[16] Cetinkaya, S., Mutlu, F. & Lee, C-Y., 2006. A comparison if outbound dispatch policies for integrated inventory and transportation decisions. European Journal of Operational Research, 17(1), 1094–1112.

[17] Chen, F., Wang, T. and Xu, T., 2005. Integrated inventory replenishment and temporal shipment consolidation: a comparison of quantity-based and time-based models. Annals of Operations Research, 135(2), 197–210.

[18] Cetinkaya, S., Tekin, E., Lee, C.-Y., 2008. A stochastic model for integrated inventory replenishment and outbound shipment release decisions. IIE Transactions, 40(3), 324–340.

[19] Hong, K.-S., Lee, C., 2013. Optimal time-based consolidation policy with price sensitive demand. International Journal of Production Economics, 143(2), 275–284.